D Kirkwood

189.2

A

COMPLETE KEY

TO

GUMMERE'S SURVEYING;

IN WHICH

THE OPERATIONS OF ALL THE EXAMPLES,

NOT SOLVED IN THAT WORK,

ARE EXHIBITED AT LARGE

PRINCIPALLY DESIGNED

TO FACILITATE THE LABOUR OF TEACHERS,

AND TO ASSIST THOSE

WHO HAVE NOT THE OPPORTUNITY OF THEIR INSTRUCTION.

BY SAMUEL ALSOP.

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KEY

TO

GUMMERE'S SURVEYING.

PLANE TRIGONOMETRY.

CASE 1.

EXAMPLE 3. (Pl. 1, fig. 1.)

Angle C=180°—A—B=46° 15′.

As sin. A 79° 23′ - - - - - - - -	Ar. Co.	0.00750
Is to sin. B 54° 22′ - - - - - - - - - -		9.90996
So is BC 125 - - - - - - - - - - -		2.09691
To AC 103.4 - - - - - - - - - - -		2.01437

Again,

As sin. A - - - - - - - - - - -	Ar. Co.	0.00750
Is to sin. C 46° 15′ - - - - - - - - - -		9.85876
So is BC - - - - - - - - - - - -		2.09691
To AB 91.87 - - - - - - - - - - -		1.96317

EXAMPLE 4. (Pl. 1, fig. 2.)

Angle C=90°—A=33° 12′.

As sin. C 33° 12′ - - - - - - - -	Ar. Co.	0.26157
Is to sin. B 90 - - - - - - - - - - -		10.00000
So is AB 53.66 - - - - - - - - - - -		1.72965
To AC 98 - - - - - - - - - - - -		1.99122

As sin. C - - - - - - - - - - -	Ar. Co.	0.26157
Is to sin. A 56° 48′ - - - - - - - - - -		9.92260
So is AB - - - - - - - - - - - -		1.72965
To BC 82 - - - - - - - - - - - -		1.91382

Example 5. (Pl. 1, fig. 2.)

Angle C=90°—A=50° 50′.

As sin. A 39° 10′	Ar. Co.	0.19957
Is to sin. B 90°		10.00000
So is BC 407.37		2.60999
To AC 645		2.80956
As sin. A	Ar. Co.	0.19957
Is to sin. C 50° 50′		9.88948
So is BC		2.60999
To AB 500.1		2.69904

CASE 2.

Example 3. (Pl. 1, fig. 1.)

As AC 306	Ar. Co.	7.51428
Is to AB 274		2.43775
So is sin. B 78° 13′		9.99075
To sin. C 61 14		9.94278
139 27		
180		
A 40 33		
As sin. B 78° 13′	Ar. Co.	0.00925
Is to sin. A 40° 33′		9.81299
So is AC 306		2.48572
To BC 203.2		2.30796

Example 4. (Pl. 1, fig. 2.)

As AC 272	Ar. Co.	7.56543
Is to AB 232		2.36549
So is sin. B 90°		10.00000
To sin. C 58° 32′		9.93092
A 31° 28′		

As sin. B	Ar. Co.	0.00000
Is to sin. A 31° 28′		9.71767
So is AC		2.43457
To BC 142		2.15224

EXAMPLE 5. (Pl. 1, fig. 2.)

As AC 150	Ar. Co.	7.82391
Is to BC 69		1.83885
So is sin. B 90°		10.00000
To sin. A 27° 23′		9.66276
C 62° 37′		

As sin. B	Ar. Co.	0.00000
Is to sin. C 62° 37′		9.94839
So is AC		2.17609
To AB 133.2		2.12448

CASE 3.

EXAMPLE 2. (Pl. 1, fig. 3.)

$$\frac{A+C}{2}=\frac{180°-B}{2}=39° 15'$$

As AB+BC 185	Ar. Co.	7.73283
Is to AB—BC 33		1.51851
So is tang. $\frac{C+A}{2}$ 39° 15′		9.91224
To tang. $\frac{C-A}{2}$ 8° 18′		9.16358
C 47° 33′		

As sin. A 30° 57′	Ar. Co.	0.28879
Is to sin. B 101° 30′		9.99119
So is BC 76		1.88081
To AC 144.8		2.16079

EXAMPLE 3. (Pl. 1, fig. 2.)

$$\frac{C+A}{2}=\frac{180°-B}{2}=45°$$

As AB+BC 1677 - - - - - - - -	Ar. Co.	6.77547
Is to AB—BC 103 - - - - - - - - - -		2.01284
So is tang. $\frac{C+A}{2}$ - - 45° - - - - - - - -		10.00000
To tang. $\frac{C-A}{2}$ - - 3° 31′ - - - - - -		8.78831
C - - 48° 31′		
As sin. A 41° 29′ - - - - - - - -	Ar. Co.	0.17888
Is to sin. B 90 - - - - - - - - - - -		10.00000
So is BC 787 - - - - - - - - - - - -		2.89597
To AC 1188 - - - - - - - - - - - -		3.07485

CASE 4.

RULE 1.

EXAMPLE 2. (Pl. 1, fig. 4.)

As AB 64 - - - - - - - - - -	Ar. Co.	8.19382
Is to AC+BC 81 - - - - - - - - - - -		1.90849
So is AC—BC 13 - - - - - - - - - - -		1.11394
To AD—BD - - 16.45 - - - - - - -		1.21625
Half difference - 8.225		
½ AB - - 32		
AD - - 40.225		
DB - - 23.775		
As AC 47 - - - - - - - - - -	Ar. Co.	8.32790
Is to AD 40.225 - - - - - - - - - - -		1.60449
So is sin. D - - 90° - - - - - - - - -		10.00000
To sin. ACD - - 58° 51′ - - - - - -		9.93239
A - - - 31° 9		

As BC 34		Ar. Co.	8.46852
Is to BD 23.775			1.37612
So is sin. D	90°		10.00000
To sin. BCD	44° 22′		9.84464
B	45 38		

Also ACB=ACD+BCD=103° 13′

EXAMPLE 3. (Pl. 1, fig. 4.)

As AB 108		Ar. Co.	7.96658
Is to AC+BC 142			2.15229
So is AC—BC 34			1.53148
To AD—BD	44.70		1.65035
Half difference	22.35		
½ AB	54		
AD	76.35		
BD	31.65		

As AC 88		Ar. Co.	8.05552
Is to AD 76.35			1.88281
So is sin. D	90°		10.00000
To sin. ACD	60° 11′		9.93833
A	29° 49′		

As BC 54		Ar. Co.	8.26761
Is to BD 31.65			1.50037
So is sin. D	90°		10.00000
To sin. BCD	35° 53′		9.76798
B	54° 7′		

ACB=ACD+DCB=96° 4′.

Rule 2.

Example 2. (Pl. 1, fig. 4.)

AC	47		
AB	64	Ar. Co.	8.19382
BC	34	Ar. Co.	8.46852
	2)145		
Half sum	72.5	- - - - -	1.86034
Difference	25.5	- - - - -	1.40654
			2)19.92922
Cos. ½ B	22° 49′	- - -	9.96461
B	45° 38′		

Example 3. (Pl. 1, fig. 4.)

AB	108		
BC	54	Ar. Co.	8.26761
AC	88	Ar. Co.	8.05552
	2)250		
Half sum	125	- - - - - -	2.09691
Difference	17	- - - - - -	1.23045
			2)19.65049
Cos. ½ C	48° 2′	- - -	9.82524
C	96° 4′		

RIGHT ANGLED TRIANGLES.

First Method.

Example 3. (Pl. 1, fig. 5.)

Making AC radius, CB is sine of A, and AB is cos. A; hence,

As radius	Ar. Co.	0.00000
Is to sin. A 27° 46′		9.66827
So is AC 36.57		1.56312
To BC 17.04		1.23139

And,

As radius	Ar. Co.	0.00000
Is to cos. A		9.94687
So is AC		1.56312
To AB 32.36		1.50999

EXAMPLE 4. (Pl. 1, fig. 5.)

Making AC radius, we have CB the sine, and AB the cosine of A; hence,

As sin. A 42° 9′	Ar. Co.	0.17323
Is to radius		10.00000
So is BC 193.6		2.28691
To AC 288.5		2.46014

And,

As sin. A	Ar. Co.	0.17323
Is to cos. A		9.87005
So is BC		2.28691
To AB 213.9		2.33019

EXAMPLE 5. (Pl. 1, fig. 6.)

Making the base AB radius, we have BC the tangent, and AC the secant of A; hence,

As AB 46.72	Ar. Co.	8.33050
Is to BC 57.9		1.76268
So is rad.		10.00000
To tang. A 51° 6′		10.09318

And,

As rad.	Ar. Co.	0.00000
Is to secant A		10.20207
So is AB		1.66950
To AC 74.4		1.87157

Second Method.—By Logarithms.

EXAMPLE 3.

Hypothenuse	403		
Base - - -	321		
Sum - - - -	724	- - - log.	2.85974
Difference - -	82	- - - - "	1.91381
			2)4.77355
Perpendicular	243.65	- - - -	2.38677

EXAMPLE 4.

Perpendicular	27.2	- - - log.	1.43457	
			2.86914	
Base - - -	31.04	- - - - -	1.49192	1.49192
	23.835	- - - - -	1.37722	
	54.875	- - - - - - -		1.73937
				2)3.23129
Hypothenuse	41.27	- - - - - - - -		1.61564

APPLICATION OF PLANE TRIGONOMETRY TO THE MENSURATION OF DISTANCES AND HEIGHTS.

EXAMPLE 1. (*See* fig. 54, *Surveying*.)

Angle C=180°—A—B=56° 23′.

To find AC:

As sin. C 56° 23′ - - - - - - - -	Ar. Co.	0.07948
Is to sin. B 49° 23′ - - - - - - - - - -		9.88029
So is AB 500 yards - - - - - - - - - -		2.69897
To AC 455.8 - - - - - - - - - - - -		2.65874

To find BC:

As sin. C	Ar. Co.	0.07948
Is to sin. A 74° 14′		9.98334
So is AB		2.69897
To BC 577.8		2.76179

EXAMPLE 2. (Fig. 55, *Surveying*.)

As BC+AC 1575	Ar. Co.	6.80272
Is to BC—AC 105		2.02119
So is tang. $\frac{A+B}{2}$ 62° 10′		10.27738
To tang. $\frac{A-B}{2}$ 7° 12′		9.10129
B 54° 58′		

As sin. B	Ar. Co.	0.08681
Is to sin. C 55° 40′		9.91686
So is AC 735		2.86629
To AB 741.2		2.86996

EXAMPLE 3. (Fig. 56, *Surveying*.)

Angle CAD=180—ADC—ACD=31° 10′

To find AC:

As sin. CAD 31° 10′	Ar. Co.	0.28607
Is to sin. ADC 53° 30′		9.90518
So is CD 300		2.47712
To AC 465.98		2.66837

Angle CBD=180—BCD—BDC=22° 55′

To find CB:

As sin. CBD 22° 55′	Ar. Co.	0.40961
Is to sin. CDB 98° 45′		9.99492
So is CD		2.47712
To CB 761.47		2.88165

To find AB:

As BC+AC 1227.45		Ar. Co.	6.91100
Is to BC—AC 295.49			2.47054
So is tang. $\frac{CAB+CBA}{2}$	71° 30′		10.47548
To tang. $\frac{CAB—CBA}{2}$	35° 44′		9.85702
CBA	35° 46′		
As sin CBA		Ar. Co.	0.23323
Is to sin. BCA 37°			9.77946
So is CA 465.98			2.66837
To AB 479.8			2.68106

EXAMPLE 4. (Fig. 57, *Surveying.*)

To find C:

AB	3		
AC	2	Ar. Co.	9.69897
BC	1.8	Ar. Co.	9.74473
	2)6.8		
Half sum	3.4		0.53148
Difference	.4		—1.60206
			2)19.57724
Cos. ½ C	52° 4′		9.78862
C	104° 8′		

To find BD:

As sin. D 17° 47′		Ar. Co.	0.51510
Is to sin. C 104° 8′			9.98665
So is BC 1.8			0.25527
To BD 5.715			0.75702

To find CD:

As sin. D	Ar. Co.	0.51510
Is to sin. DBC 58° 5′		9.92881
So is BC		0.25527
To CD 5.002		0.69918

EXAMPLE 5. (Fig. 58, *Surveying*.)

To find BAC:

BC	7.2		
AB	12	Ar. Co.	8.92082
AC	8	Ar. Co.	9.09691
	27.2		
Half sum	13.6		1.13354
Difference	6.4		0.80618
			2)19.95745
Cos. ½ BAC	17° 47′½		9.97872
BAC	35° 35′		

To find AE:

As sin. AEB 136°	Ar. Co.	0.15823
Is to sin. EBA 19°		9.51264
So is AB 12		1.07918
To AE 5.624		0.75005

To find ACE:

As AC+AE 13.624	Ar. Co.	8.86569
Is to AC—AE 2.376		0.37585
So is tang. $\frac{AEC+ACE}{2}$ - 84° 42′½		11.03329
To tang. $\frac{AEC—ACE}{2}$ - 62 1½		10.27483
AEC - 146 44		
ACE - 22° 41′		

To find AD:

As sin. ADC 19°	Ar. Co.	0.48736
Is to sin. ACD 22° 41′		9.58618
So is AC 8		0.90309
To AD 9.476		0.97663

To find CD:

As sin. ADC	Ar. Co.	0.48736
Is to sin. DAC 138° 19′		9.82283
So is AC		0.90309
To DC 16.34		1.21328

To find BD:

As sin. ADB 44°	Ar. Co.	0.15823
Is to sin. BAD 102° 44′		9.98919
So is AB 12		1.07918
To DB 16.85		1.22660

EXAMPLE 6. (Fig. 59, *Surveying*.)

To find ABC:

AC	46		
AB	50	Ar. Co.	8.30103
BC	40	Ar. Co.	8.39794
	2)136		
Half sum	68		1.83251
Difference	22		1.34242
			2)19.87390
Cos. ½ ABC	30° 8′		9.93695
ABC	60° 16′		

To find CD and CE:

As sin. ADC 60° 16'	Ar. Co.	0.06131
Is to radius		10.00000
So is AC 46		1.66276
To CD 52.98		1.72407

CE = ½ CD = 26.49

Also CAE = 90° — ADC = 90° — ABC = 29° 44'.

EXAMPLE 7. (Fig. 61, *Surveying*.)

Making DE radius, EC is tangent of D; hence,

As radius	Ar. Co.	0.00000
Is to tang. D 47° 30'		10.03795
So is DE 100		2.00000
To EC 109.13		2.03795
EB 5		
BC 114.13		

EXAMPLE 8. (Fig. 62, *Surveying*.)

To find DC:

As sin. ACD 25°	Ar. Co.	0.37405
Is to sin. CAD 26° 30'		9.64953
So is AD 75 ft.		1.87506
To DC 79.18 ft.		1.89864

To find BC:

As radius	Ar. Co.	0.00000
Is to sin. CDB 51° 30'		9.89354
So is CD		1.89864
To CB 61.97 ft.		1.79218

EXAMPLE 9. (Fig. 63, *Surveying*.)

To find DC:

As sin. ACD 23° 50'	Ar. Co.	0.39354
Is to sin. CAD 44°		9.84177
So is AD 134		2.12710
To DC 230.4		2.36241

To find CE:

As sin. CED 141°	Ar. Co.	0.20113
Is to sin. CDE 16° 50′		9.46178
So is CD		2.36241
To CE 106		2.02532

To find EB:

As radius	Ar. Co.	0.00000
Is to sin. CDB 67° 50′		9.96665
So is CD		2.36241
To CB 213.3		2.32906

CE 106

BE 107.3

EXAMPLE 10. (Fig. 64, *Surveying*.)

To find BD:

As sin. CDB 17° 15′	Ar. Co.	0.52791
Is to sin. BCD 23° 45′		9.60503
So is CB 60		1.77815
To BD 81.49		1.91109

To find AD:

As BD+BA 121.49	Ar. Co.		7.91546
Is to BD—BA 41.49			1.61794
So is tang. $\frac{BAD+BDA}{2}$	69° 30′		10.42726
To tang. $\frac{BAD—BDA}{2}$	42° 25′		9.96066
BDA	27° 5′		

As sin. ADB 27° 5′	Ar. Co.	0.34172
Is to sin. ABD 41°		9.81694
So is AB 40		1.60206
To AD 57.64		1.76072

EXAMPLE 11. (Fig. 65, *Surveying.*)

To find AD:

As sin. CAD 27°	Ar. Co.	0.34295
Is to sin. ACD 138°		9.82551
So is CD 132		2.12057
To AD		2.28903

To find AB:

As sin. ABD 109°	Ar. Co.	0.02433
Is to sin. ADB 8°		9.14356
So is AD		2.28903
To AB 28.64		1.45692

EXAMPLE 12. (Fig. 66, *Surveying.*)

To find C:

As BC 100	Ar. Co.	8.00000
Is to AB 34		1.53148
So is rad.		10.00000
To tan. C 18° 47′		9.53148

To find AE=EC:

As sin. AEB 37° 34′	Ar. Co.	0.21490
Is to radius		10.00000
So is AB 34		1.53148
To AE 55.77		1.74638

100

BE 44.23

Otherwise.

$AC^2=AB^2+BC^2=11156.$

And since the triangles CDE and CBA are similar, we have CB : CA : : CD ($\frac{1}{2}$ CA) : CE.

whence $CE=\frac{CA^2}{2\,CB}=\frac{11156}{200}=55.78$

and $BE=CB-CE=44.22$

PRACTICAL QUESTIONS.

EXAMPLE 1. (Pl. 1, fig. 2.)

Making AB radius, BC is tangent of A.

As radius	Ar. Co.	0.00000
Is to tang. A 52° 30′		10.11502
So is AB 85		1.92942
To BC 110.8		2.04444

EXAMPLE 2. (Pl. 1, fig. 2.)

Make AB radius, then will BC be the tangent and AC the secant of A; hence,

As radius	Ar. Co.	0.00000
Is to tang. A 61° 45′		10.26977
So is AB 73		1.86332
To BC 135.9		2.13309

And,

As radius	Ar. Co.	0.00000
Is to secant A		10.32485
So is AB		1.86332
To AC 154.2		2.18817

EXAMPLE 3. (Pl. 1, fig. 7.)

To find BD. We have in the triangle ABD, the angles and side AB. Hence,

As sin. ADB 31°	Ar. Co.	0.28816
Is to sin. BAD 100°		9.99335
So is AB 339		2.53020
To BD 648.2		2.81171

Again, in ABC we have the angles, and side AB, to find BC. Thus,

As sin. ACB 22° 30′ - - - - - -	Ar. Co.	0.41716
Is to sin. BAC 36° 30′ - - - - - - - - - -		9.77439
So is AB 339 - - - - - - - - - - - -		2.53020
To BC 526.9 - - - - - - - - - - - -		2.72175

In DBC we have the sides DB and BC, and included angle DBC= 72°. To find the side DC. Thus.

$$BCD+BDC=180°-72°=108°.$$

Then,

As BD+BC 1175.1 - - - - - - -	Ar. Co.	6.92992
Is to BD—BC 121.3 - - - - - - - - - -		2.08386
So is tang. $\frac{BCD+BDC}{2}$ - 54° - - - - - -		10.13874
To tang. $\frac{BCD-BDC}{2}$ - 8° 5′ - - - - -		9.15252
BDC 45° 55′		

And,

As sin. BDC 45° 55′ - - - - - - -	Ar. Co.	0.14368
Is to sin. DBC 72° - - - - - - - - - -		9.97821
So is BC - - - - - - - - - - - -		2.72175
To CD 697.65 - - - - - - - - - - - -		2.84364

This example might have been solved by finding AD=496.76, AC=759.33; whence the angle ADC would be found to be 76° 55′, and CD=697.65, as before.

Example 4. (Pl. 1, fig. 8.)

Construction.

With the given distances construct the triangle ABC. Make ACE and CAE respectively equal to 13° 30′ and 29° 50′. About the triangle AEC describe the circle ACD. Join EB, and produce it to meet the circumference in D, which will be the situation of the observer.

Since the angles ADE and ACE are subtended by the same arc, we have ADE=ACE=13° 30′. Also CDE=CAE=29° 50′.

Calculation.

In the triangle ABC, we have the three sides to find the angle BAC. Thus,

BC	262		
AC	404	Ar. Co.	7.39362
AB	213	Ar. Co.	7.67162
	2)879		
Half sum	439.5	- - - - -	2.64296
Difference	177.5	- - - - -	2.24920
			2)19.95740
Cos. ½ BAC	17° 48′	- - -	9.97870
BAC	35° 36′		

In the triangle ACE we have the angles and side AC, to find AE.

As sin. AEC	136° 40′	- - - - -	Ar. Co. 0.16352
Is to sin. ACE	13° 30′	- - - - - - - - -	9.36819
So is AC	404	- - - - - - - - - - - -	2.60638
To AE	137.43	- - - - - - - - - - - -	2.13809

In the triangle ABE we have the sides AB and AE, and the included angle BAE=BAC+CAE=65° 26′. To find ABE, thus:

As AB+AE	350.43	- - - - - - -	Ar. Co. 7.45540
Is to AB—AE	75.57	- - - - - - - - -	1.87835
So is tang. $\frac{AEB+ABE}{2}$	57° 17′	- - - -	10.19219
To tang. $\frac{AEB-ABE}{2}$	18° 33′	- - - -	9.52594
	ABE=38° 44′		

In ABD we have ABD=180°—38° 44′=141° 16′, ADB=13° 30′, and BAD=38° 44′—13° 30′=25° 14′. To find AD and DB:

As sin. ADB	13° 30′	- - - - -	Ar. Co. 0.63181
Is to sin. BAD	25° 14′	- - - - - - - - -	9.62972
So is AB	213	- - - - - - - - - - - -	2.32838
To BD	389	- - - - - - - - - - - -	2.58991

And,

As sin. ADB 13° 30′ - - - - -	Ar. Co.	0.63181
Is to sin. ABD 141° 16′ - - - - - - - -		9.79636
So is AB 213 - - - - - - - - - - - -		2.32838
To AD 570.9 - - - - - - - - - - - -		2.75655

Finally, in ADC we have the angle ADC=43° 20′, CAD=BAC+BAD=60° 50′ and the side AC; to find CD. Thus,

As sin. ADC 43° 20′ - - - - - -	Ar. Co.	0.16352
Is to sin. CAD 60° 50′ - - - - - - - -		9.94112
So is AC 404 - - - - - - - - - - -		2.60638
To CD 514.1 - - - - - - - - - - -		2.71102

This might have been solved by finding ACB = 28° 14′, CE = 292.87, whence CBE would have been found to be = 77° 26′, BD = 388.9, DC = 514, and AD = 570.8.

EXAMPLE 5. (Pl. 1, Fig. 9.)

Here $AD = \sqrt{BD^2 - AB^2} = \sqrt{1296} = 36$.

And $AC = AD + DC = 75$ Ans.

Or, Trigonometrically;

As BD 39 - - - - - - - - - -	Ar. Co.	8.40894
Is to BA 15 - - - - - - - - - - -		1.17609
So is radius - - - - - - - - - - -		10.00000
To cos. B 67° 23′ - - - - - - - - -		9.58503

And,

As radius - - - - - - - - - -	Ar. Co.	0.00000
Is to sin. B 67° 23′ - - - - - - - - -		9.96525
So is BD 39 - - - - - - - - - - -		1.59106
To AD 36 - - - - - - - - - - -		1.55631

AC - 75

EXAMPLE 6. (Pl. 1, fig. 10.)

The angle ACB = DBC — DAC = 25°.

Then,

As sin ACB 25°	Ar. Co.	0.37405
Is to sin. BAC 26° 30′		9.64953
So is AB 75		1.87506
To BC 79.18		1.89864

To find CD, and BD:

As radius	Ar. Co.	0.00000
Is to sin. B 51° 30′		9.89354
So is CB		1.89864
To CD 61.97		1.79218

And,

As radius	Ar. Co.	0.00000
Is to cos. B		9.79415
So is CB		1.89864
To BD 49.29		1.69279

EXAMPLE 7. (Pl. 1, fig. 11.)

Here ACB = CAD = 35° and BAC = 55°

Hence,

As rad.	Ar. Co.	0.00000
Is to tan. BAC 55°		10.15477
So is AB 143.		2.15534
To BC 204.2		2.31011

EXAMPLE 8. (Pl. 1, fig. 12.)

Construction.

Make AB=76, the distance from the lower column to the statue's base. Erect the perpendiculars AD and BF, making the former= 50. With D as a centre and distance 86, cross BF in F, which will be the head of the statue.

Make AI = 64, draw IE parallel to AC, with F as a centre and distance 97, cross IE in E, then EC perpendicular to AC, will be the higher column.

Calculation.

To find FDG and side DG:

As DF 86	Ar. Co.	8.06550
Is to FG 76		1.88081
So is radius		10.00000
To sin. FDG 62° 5½′		9.94631
As radius	Ar. Co.	0.00000
Is to cos. FDG 62° 5½′		9.67030
So is FD 86		1.93450
To DG 40.25		1.60480

To find EFH and FH, we have FE = 97 and EH = GI = GD + DI = 54.25. Hence,

As EF 97	Ar. Co.	8.01323
Is to EH 54.25		1.73440
So is radius		10.00000
To sin. EFH 34°		9.74763

And,

As radius	Ar. Co.	0.00000
Is to cos. F 34°		9.91857
So is EF 97		1.98677
To FH 80.42		1.90534

To find ED, we have EI = HF + FG = 156.42 and DI = 14. Hence,

As IE 156.42	Ar. Co.	7.80571
Is to ID 14		1.14613
So is radius		10.00000
To tan. IED 5° 7′		8.95184

As rad. - - - - - - - - - - - Ar. Co.	0.00000
Is to sec. E 5° 7′ - - - - - - - - - - -	10.00173
So is IE 156.42 - - - - - - - - - - -	2.19429
To ED 157.04 - - - - - - - - - - -	2.19602

Otherwise.

$GD = \sqrt{FD^2 - FG^2} = \sqrt{1620} = 40.25.$

$GI = GD + DI = 54.25.$

$FH = \sqrt{FE^2 - EH^2} = \sqrt{6465.9375} = 80.41.$

$IE = FH + FG = 156.41.$

$DE = \sqrt{IE^2 + ID^2} = \sqrt{24660.0881} = 157.03.$

SURVEYING.

CHAPTER I.

DIMENSIONS OF A SURVEY.

PROBLEM 8.

EXAMPLE 2.

Angle B = 34°+35° = 69°.

EXAMPLE 3.

Here the first bearing must be reversed, since it is towards the station C. It becomes N. 35° W. Hence C=180°—(35°+87°)=58°.

EXAMPLE 4.

D = 180° — (87° — 58°) = 151°.

PROBLEM 9.

EXAMPLE 2.

1st side	S. 40½° E.	3d	N. 29¼° E.
	N. 54 E.		N. 54 E.
	94½		N. 24¾ W.
	180		
	N. 85½ E.		
4th	N. 28¾° E.	5th	N. 57° W.
	N. 54 E.		N. 54 E.
	N. 25¼ W.		111
			180
			S. 69 W.
6th	S. 47° W.		
	N. 54 E.		
	S. 7 E.		

EXAMPLE 3.

1st	S. 45½°	W.	2d	N. 50°	W.
	S. 20½	W.		S. 20½	W.
	S. 25	W.		N. 70½	W.
3d	N. 0°	W.	4th	N. 85°	E.
	S. 20½	W.		S. 20½	W.
	N. 20½	W.		N. 64½	E.
5th	S. 47°	E.	7th	N. 51½°	W.
	S. 20½	W.		S. 20½	W.
	S. 67½	E.		N. 71½	W.

PROBLEM 10.

EXAMPLE 1.

As radius - - - - - - - - - -	Ar. Co.	0.00000
Is to cos. bearing 53° 20′ - - - - - - - -		9.77609
So is distance 13.25 - - - - - - - - -		1.12222
To difference of latitude 7.912 N. - - - -		0.89831

And,

As radius - - - - - - - - - -	Ar. Co.	0.00000
Is to sin. bearing - - - - - - - - - -		9.90424
So is distance - - - - - - - - - -		1.12222
To departure 10.63 E. - - - - - - - -		1.02646

EXAMPLE 2.

As radius - - - - - - - - - -	Ar. Co.	0.00000
Is to cosecant bearing 32° 30′ - - - - -		10.26978
So is departure 10.96 - - - - - - - -		1.03981
To distance 20.40 - - - - - - - - -		1.30959

And,

As radius - - - - - - - - - -	Ar. Co.	0.00000
Is to cotangent bearing - - - - - - - -		10.19581
So is departure - - - - - - - - -		1.03981
To difference of latitude 17.20 - - - - -		1.23562

EXAMPLE 3.

As difference of latitude 34.43 - -	Ar. Co.	8.46306
Is to distance 44 - - - - - - - - - - -		1.64345
So is radius - - - - - - - - - - - -		10.00000
To secant of bearing 38° 30′ - - - - - - -		10.10651

And,

As rad. - - - - - - - - - - -	Ar. Co.	0.00000
Is to tang. bearing 38° 30′ - - - - - - -		9.90061
So is diff. lat. 34.43 - - - - - - - - - -		1.53694
To departure 27.39 - - - - - - - - - -		1.43755

EXAMPLE 4.

As radius - - - - - - - - - -	Ar. Co.	0.00000
Is to secant of bearing 32° 30′ - - - - - -		10.07397
So is diff. of lat. 17.21 - - - - - - - -		1.23578
To distance 20.41 - - - - - - - - - -		1.30975

And,

As radius - - - - - - - - - -	Ar. Co.	0.00000
Is to tang. bearing 32° 30′ - - - - - - -		9.80419
So is diff. latitude 17.21 - - - - - - - -		1.23578
To departure 10.96 - - - - - - - - - -		1.03997

EXAMPLE 5.

As diff. of lat. 27.92 N. - - - -	Ar. Co.	8.55408
Is to departure 5.32 E. - - - - - - - - -		0.72591
So is radius - - - - - - - - - - - -		10.00000
To tang. bear. 10° 47′ - - - - - - - - -		9.27999

And,

As radius - - - - - - - - - -	Ar. Co.	0.00000
Is to secant of bearing 10° 47′ - - - - - -		10.00774
So is diff. of lat. - - - - - - - - - - -		1.44592
To dist. 28.42 - - - - - - - - - - -		1.45366

EXAMPLE 6.

As distance 35.35 - - - - - - - Ar. Co. 8.45161
Is to departure 15.08 - - - - - - - - - - 1.17840
So is radius - - - - - - - - - - - - - 10.00000

To sin. bearing 25° 15′ - - - - - - - - 9.63001

And,

As radius - - - - - - - - - - - Ar. Co. 0.00000
Is to cos. bearing 25° 15′ - - - - - - - - 9.95639
So is distance - - - - - - - - - - - - - 1.54839

To diff. of lat. 31.97 - - - - - - - - - 1.50478

PROBLEM 12.

Sta.	Courses.	Dist.	N.	S.	E.	W.	Cor. N.	Cor. E.	N.	S.	E.	W.
1	N. 75 E.	13.70	3.54		13.24		2	2	3.56		13.26	
2	N. 20½ E.	10.30	9.65		3.61		1	1	9.66		3.62	
3	East.	16.20			16.20		2	2	.02		16.22	
4	S. 33½ W.	35.30		29.44		19.49	5	5		29.39		19.44
5	S. 76 W.	16.00		3.87		15.52	2	2		3.85		15.50
6	North.	9.00	9.00				1	1	9.01		.01	
7	S. 84 W.	11.60		1.21		11.54	2	2		1.19		11.52
8	N. 53¼ W.	11.60	6.94			9.29	2	2	6.96			9.27
9	N. 36¾ E.	19.36	15.51		11.59		3	2	15.54		11.61	
10	N. 22½ E.	14.00	12.93		5.36		2	2	12.95		5.38	
11	S. 76¾ E.	12.00		2.75	11.68		2	2		2.73	11.70	
12	S. 15 W.	10.85		10.48		2.81	2	1		10.46		2.80
13	S. 18 W.	10.62		10.10		3.28	2	1		10.08		3.27
			57.57	57.85 57.57	61.68	61.93 61.68						

Error South .28 .25 Error West.

CHAPTER II.

SUPPLYING OMISSIONS.

PROBLEM I.

EXAMPLE 2.

Sta.	Courses.	Dist.	N.	S.	E.	W.
1	N. 15¾° W.	9.40	9.05			2.55
2	N. 63¾ E.	10,43	4.61		9.36	
3	S. 49 E.	8.12		5.33	6.13	
4	S. 13½ E.	8.45		8.22	1.98	
5	S. 16¾ E.	6.44		6.17	1.86	
6				(6.11)		(10.64)
7	N. 60 W.	9.72	4.86			8.41
8	N. 17¼ W.	7.65	7.31		2.27	
			25.83	25.83	21.60	21.60

Then,

As diff. lat. 6.11 S. - - - - - - - Ar. Co. 9.21396
Is to depart. 10.64 W. - - - - - - - - - 1.02694
So is radius - - - - - - - - - - - - - 10.00000

To tang. bearing S. 60° 8′ W. - - - - - - 10.24090

And, As radius - - - - - - - - - - Ar. Co. 0.00000
Is to secant bearing 60° 8′ - - - - - - - 10.30279
So is diff. lat. - - - - - - - - - - - - 0.78604

To distance 12.27 - - - - - - - - - - 1.08883

EXAMPLE 3.

Sta.	Courses.	Dist.	N.	S.	E.	W.
1	S. 52° W.	10.70		6.59		8.43
2	S. 7½ W.	13.92		13.80		1.82
3	S. 34¼ E.	9.00		7.44	5.07	
4			(27.83)		(5.18)	
				27.83	10.25	10.25

Then,

As diff. lat. 27.83 - - - - - - - -	Ar. Co.	8.55549
Is to departure 5.18 - - - - - - - - - -		0.71433
So is radius - - - - - - - - - - - - -		0.00000
To tang. bearing N. 10° 33′ E. - - - - - -		9.26982

And,

As radius - - - - - - - - - -	Ar. Co.	0.00000
Is to secant bearing 10° 33′ - - - - - -		10.00740
So is diff. lat. 27.83 - - - - - - - - - -		1.44451
To distance 28.31 - - - - - - - - - - -		1.45191

EXAMPLE 4.

Sta.	Bearing.	Dist.	N.	S.	E.	W.
1	S. 10° E.	92.20		90.80	16.01	
2	S. 15 W.	120.50		116.39		31.19
3	S. 18½ W.	205.00		194.40		65.05
4	S. 71½ E.	68.00		21.58	64.49	
5						
				423.17	80.50	96.24
						80.50
						15.74

Then,

As diff. of latitude 423.17 - - - -	Ar Co.	7.37348
Is to departure 15.74 - - - - - - - -		1.19700
So is radius - - - - - - - - - - - -		10.00000
To tang. bearing 2° 8′ - - - - - - - -		8.57048

And,

As radius - - - - - - - - - -	Ar. Co.	0.00000
Is to secant bearing 2° 8′ - - - - - - -		10.00030
So is diff. lat. 423.17 - - - - - - - -		2.62652
To distance 423.47 - - - - - - - - -		2.62682

PROBLEM II.

EXAMPLE 2.

Sta.	Bearing.	Changed Bearing.	Dist.	N.	S.	E.	W.
1	S. 40½ E.	N. 85½ E.	31.80	2.49		31.70	
2	N. 54 E.	North.		(2.08)			
3	N. 29¼ E.	N. 24¾ W.	2.21	2.01			.93
4	N. 28¾ E.	N. 25¼ W.	35.35	31.98			15.08
5	N. 57 W.	S. 69 W.			(7.49)		(19.51)
6	S. 47 W.	S. 7 E.	31.30		31.07	3.82	
				38.56	38.56	35.52	35.52

As radius - - - - - - - - - - Ar. Co. 0.00000
Is to cosec. changed bearing 69° - - - - - 10.02985
So is departure 19.51 - - - - - - - - - 1.29026
To distance 5th side 20.90 - - - - - - - 1.32011

And, As radius - - - - - - - - - - Ar. Co. 0.00000
Is to cotang bearing 69° - - - - - - - - 9.58418
So is departure 19.51 - - - - - - - - - 1.29026
To diff. latitude 7.49 S. - - - - - - - - 0.87444

PROBLEM III.

EXAMPLE 2.

Sta.	Bearing.	Changed Bearing.	Dist.	N.	S.	E.	W.
1	S. 40½ E.	N. 85½ E.	31.80	2.49		31.70	
2	N. 54 E.	North.		(2.09)			
3	N. 29¼ E.	N. 24¾ W.	2.21	2.01			.93
4	N. E.		35.35	(31.97)			(15.08)
5	N. 57 W.	S. 69 W.	20.90		7.49		19.51
6	S. 47 W.	S. 7 E.	31.30		31.07	3.82	
				38.56	38.56	35.52	35.52

Then,

As distance 4th side 35.35 - - -	Ar. Co.	8.45161
Is to departure 15.08 - - - - - - - - -		1.17840
So is radius - - - - - - - - - -		10.00000
To sine chang. bearing N. 25° 15′ W. - -		9.63001
54		
Bearing 4th side N. 28° 45′ E.		

And,

As radius - - - - - - - - - -	Ar. Co.	0.00000
Is to cos. chang. bearing 25° 15′ - - - -		9.95639
So is distance - - - - - - - - - - -		1.54839
To diff. latitude 31.97 - - - - - - - -		1.50478

PROBLEM IV.

EXAMPLE 2. (Pl. 1, fig. 13.)

	Bearing.	Dist.	N.	S.	E.	W.
FA	S. E.	31.80				
AB	N. 54 E.	2.08	1.23		1.68	
BC	N. 29¼ E.	2.21	1.92		1.08	
CD	N. 28¾ E.	35.35	31.00		17.00	
DE	N. 57 W.	20.90	11.38			17.52
EF	S. W.	31.30				
Diff. latitude of EA			45.53		19.76	17.52
					17.52	
				Departure of EA	2.24	

Then, As diff. lat. EA 45.53 - - - -	Ar. Co.	8.34170
Is to departure 2.24 - - - - - - - -		0.35025
So is radius - - - - - - - - - - -		10.00000
To tang. bearing EA 2° 49′ - - - - -		8.69195

And,

As radius	Ar. Co.	0.00000
Is to secant bearing 2° 49′		10.00052
So is diff. lat.		1.65830
To distance EA 45.58		1.65882

To find AEF:

AF	31.80		
AE	45.58	Ar. Co.	8.34123
EF	31.30	Ar. Co.	8.50446
	2)108.68		
Half sum	54.34		1.73512
Difference	22.54		1.35295
			2)19.93376
Cos. ½ AEF	22° 6′		9.96688
AEF	44° 12′		
Bearing of EA	2° 49′		
" EF	S. 47° 1′ W.		

To find EAF and bearing of FA:

As AF 31.80		Ar. Co.	8.49757
Is to EF 31.30			1.49554
So is sin. AEF	44° 12′		9.84334
To sin. EAF	43° 20′		9.83645
Bearing of EA	2° 49′		
" AF	40° 31′		

CHAPTER III.

CONTENT OF LAND.

PROBLEM I.

EXAMPLE 4.

Here, Area = 176.4 × 176.4 = 31116.96 Sq. Perches,
= 194 A. 1 R. 36.96 P.

EXAMPLE 5.

Here, Area = $52.25 \times 38.24 = 1998.04$ Sq. Ch.
= 199 A. 3 R. 8.64 P.

EXAMPLE 6.

Here, Area = $16.54 \times 12.37 = 204.5998$ Sq. Ch.
= 20 A. 1 R. 33.5968 P.

EXAMPLE 7.

Here, Area = $21.16 \times 11.32 = 239.5312$ Sq. Ch.
= 23 A. 3 R. 32.4992 P.

PROBLEM 2.

EXAMPLE 2

Here, Area = $\frac{18.37 \times 13.44}{2} = \frac{246.8928}{2} = 123.4464$ Sq. Ch.
= 12 A. 1 R. 15.1424 P.

EXAMPLE 3.

Here, Area = $\frac{49 \times 34}{2} = \frac{1666}{2} = 833$ Sq. Pe.
= 5 A. 0 R. 33 Pe.

PROBLEM 3.

EXAMPLE 2. (Pl. 1, fig. 1.)

As radius - - - - - - - - - -		Ar. Co.	0.00000
Is to sin. A 47° 30′ - - - - - - - - - -			9.86763
So is AB × AC	AB 15.36 - - - - - - -		1.18639
	AC 11.46 - - - - - - -		1.05918
To double area	129.78 - - - - - - - - -		2.11320

ABC - 64.89 Ch. = 6 A. 1 R. 38. 24 P

EXAMPLE 3. (Pl. 1, fig. 14.)

Here, As radius		Ar. Co.	0.00000
Is to sin. A 66° 30′			9.96240
So is AB×AC	AB 13.84		1.14114
	AC 18.23		1.26079
To 2 ABC	231.38		2.36433

ABC - 115.69 Ch. = 11 A. 2 R. 11.04 P.

EXAMPLE 4. (Pl. 1, fig. 15.)

Here, As radius		Ar. Co.	0.00000
Is to sin. A 121° 45′			9.92960
So is AB, AC	AB 19.74		1.29535
	AC 17.34		1.23905
To 2 ABC	291.07		2.46400

ABC - 145.535 Ch. = 14 A. 2 R. 8.56 P.

PROBLEM 4.

EXAMPLE 2. (Pl. 1, fig. 1.)

Here, Angle C = 180—(A+B) = 43°. Hence,

As rad., sin. C	radius	Ar. Co.	0.00000
	sin. C 43°	Ar. Co.	0.16622
Is to sin. A, sin. B	sin. A 63°		9.94988
	sin. B 74°		9.98284
So is AB^2	AB 24.32		1.38596
	AB		1.38596
To 2 ABC	742.8		2.87086

ABC - 371.4 Ch. = 37 A. 0 R. 22.4 P.

EXAMPLE 3.

Here, the angle C = 94° 15′. Hence,

As rad., sin. C	rad.	Ar. Co.	0.00000
	sin. C 94° 15′	Ar. Co.	0.00120
Is to sin. A, sin. B	sin. A 37° 30′		9.78445
	sin. B 48° 15′		9.87277
So is AB^2	AB 17.36		1.23955
	AB		1.23955
To 2 ABC	137.25		2.13752

ABC - 68.625 Ch. = 6 A. 3 R. 18 P.

PROBLEM 5.

EXAMPLE 2.

Here, $10.64+12.28+9.00 = 31.92$ = sum of sides.

Half sum	15.96	log.	1.20303
Remainders	5.32		0.72591
	3.68		0.56585
	6.96		0.84261
			2)3.33740
Area	10)46.63 Ch.		1.66870

4.663 = 4 A. 2 R. 26.08 P.

EXAMPLE 3.

Here, $20+30+40 = 90$ = sum of sides.

Half sum	45		1.65321
Remainders	25		1.39794
	15		1.17609
	5		0.69897
			2)4.92621
	10)290.47		2.46310

29.047 A. = 29 A. 0 R. 7.52 P.

PROBLEM 6.

EXAMPLE 2.

Here, $16.10 \times \frac{6.80+3.40}{2} = 16.1 \times 5.1 = 82.11$ Ch.

= 8 A. 0 R. 33.76 P.

EXAMPLE 3.

Here, $24 \times \frac{8.27+12.43}{2} = 24 \times 10.35 = 248.4$ Ch.

= 24 A. 3 R. 14.4 P.

PROBLEM 7.

EXAMPLE 2. (Pl. 1, fig. 16.)

Here BEA = 180° — (A+B) = 34°. Hence,

As rad., sin. E	radius - -	Ar. Co.	0.00000
	sin. E 34° -	" "	0.25244
Is to sin. C, sin. D.	sin. C 120° - - - -		9.93753
	sin. D 94° - - - - -		9.99894
So is CD^2	CD 11 Ch. - - - -		1.04139
	CD - - - - - - - -		1.04139
To 2 CDE 186.93 - - - - - - - - - -			2.27169

And,

As rad., sin. E	radius - - -	Ar. Co.	0.00000
	sin. E 34° -	" "	0.25244
Is to sin. A, sin. B	sin. A 65° - - - -		9.95728
	sin. B 61° - - - -		9.99462
So is AB^2	AB 20 Ch. - - -		1.30103
	AB - - - - - - - -		1.30103
To 2 ABE - 640.33 - - - - - - - -			2.80640
2 CDE - 186.93			
2 ABCD 2)453.40			
ABCD 226.7 Ch. = 22 A. 2 R. 27.2 P.			

EXAMPLE 3. (Pl. 2, fig. 1.)

Here the angles are as follow, viz.
A = 111° 30′, B = 45°, C = 125°, D = 78° 30′, and E = 23° 30′.
Hence,

As rad., sin. E	radius - - -	Ar. Co.	0.00000
	sin. E 23° 30′	" "	0.39930
Is to sin. C, sin. D.	sin. C 125° - - - -		9.91336
	sin. D 78° 30′ - - -		9.99119
So is CD^2	CD 12.90 Ch. - - -		1.11059
	CD - - - - - - - -		1.11059
To 2 CDE 334.99 - - - - - - - - - -			2.52503

And, As rad. sin. E	radius	- - -	Ar. Co.	0.00000
	sin. E	23° 30	" "	0.39930
Is to sin. A, sin. B	sin. A	111° 30′	- - -	9.96868
	sin. B	45°	- - - -	9.84949
So is AB^2	AB	23.20 Ch.	- - -	1.36549
	AB	- - - - - - - -		1.36549
To 2 ABE	888.08	- - - - - - - - - -		2.94845
2 CDE	334.99			
	2)553.09			
	276.545 Ch. = 27 A. 2 R. 24.72 P.			

PROBLEM 8.

EXAMPLE 2. (Pl. 2, fig. 2.)

Here, As rad.	- - - - - - - - - -		Ar. Co.	0.00000
Is to sin. A	56°	- - - - - - - - - -		9.91857
So is AB.AD	AD	6.15	- - - - - - -	0.78888
	AB	8.46	- - - - - - -	0.92737
To first quantity	43.13	- - - - - - - -		1.63482
And, As rad.	- - - - - - - - - -		Ar. Co.	0.00000
Is to sin. B	98° 30′	- - - - - - - - - -		9.99520
So is AB.BC	AB	8.46	- - - - - - - -	0.92737
	BC	7.00	- - - - - - - -	0.84510
To second quantity	58.57	- - - - - - - -		1.76767
As rad.	- - - - - - - - - -		Ar. Co.	0.00000
Is to sin. 180° — (A+B)	25° 30′	- - - - -		9.63398
So is AD.BC	AD	6.15	- - - - - - -	0.78888
	BC	7.00	- - - - - - -	0.84510
To third quantity	18.53	- - - - - - - -		1.26796
First - - - -	43.13			
Second - - -	58.57			
	101.70			
	2)83.17			
	ABCD = 41.585 Ch. = 4 A. 0 R. 25.36 P			

EXAMPLE 3. (Pl. 2, fig. 3.)

Here, Angle B = 94° 30′ & C = 118° 45.

Therefore,

As radius		Ar. Co.	0.00000
Is to sin. B 94° 30′			9.99866
So is AB, BC	AB 17.53		1.24378
	BC 10.80		1.03342
First quantity	188.74		2.27586
As radius		Ar. Co.	0.00000
Is to sin. C 118° 45′			9.94286
So is BC, CD	BC 10.80		1.03342
	CD 12.92		1.11126
Second quantity	122.33		2.08754
As radius		Ar. Co.	0.00000
Is to B+C—180 33° 15′			9.73901
So is AB, CD	AB 17.53		1.24378
	CD 12.92		1.11126
Third quantity	124.18		2.09405

Second 122.33
First 188.74

2)435.25

217.625 Ch. = 21 A. 3 R. 2 P.

PROBLEM 9.

EXAMPLE 2.

$$\text{Here, Area} = \frac{12.41+8.22}{2} \times 5.15 = 53.12225 \text{ Ch.}$$

$$= 5 \text{ A. } 1 \text{ R. } 9.956 \text{ P.}$$

EXAMPLE 3.

$$\text{Here, Area} = \frac{11.34+18.46}{2} \times 13.25 = 197.425 \text{ Ch.}$$

$$= 19 \text{ A. } 2 \text{ R. } 38.8 \text{ P.}$$

PROBLEM 11.

EXAMPLE 3.

Sta.	Bearing.	Dist.	N.	S.	E.	W.	Cor. S.	Cor. E.	N.	S.	E.	W.	E. D. D.	W. D. D.	Mult.	N. Area.	S. Area.
1	S. 35¼ W.	11.20		9.14		6.47	1	1		9.15		6.46		35.61			
2	N. 45 W.	24.36	17.22			17.22	1	2	17.21			17.20		23.66	23.66 W.		407.1886
3	N. 15½ E.	10.80	10.41		2.88		0	1	10.41		2.89			14.31	37.97 W.		395.2677
4	S. 77 E.	16.00		3.60	15.59		1	1		3.61	15.60		18.49		19.48 W.	70.3228	
5	N. 87½ E.	21.50	.94		21.48		1	1	.93		21.49		37.09		17.61 E.	16.3773	
6	S. 60 E.	14.80		7.40	12.81		1	1		7.41	12.82		34.31		51.92 E.		384.7272
7	South.	10.91		10.91			1	1		10.92	.01		12.83		64.75 E.		707.0700
8	N. 85 W.	29.28	2.55			29.17	1	2	2.54			29.15		29.14	35.61 E.	90.4494	
		138.85	31.12 31.05	31.05	52.76	52.86 52.76	7	10								177.1495	1894.2535 177.1495
			Error N	.07			.10	Error W									2)1717.1040
																	A. 85,8.5520 4
																	R. 3.42080 40
																	P. 16.83200

Area 85 A. 3 R. 16.832 P.

EXAMPLE 4.

Sta.	Bearing.	Dist.	N.	S.	E.	W.	Cor. S.	Cor. E.	N.	S.	E.	W.	E. D. D.	W. D. D.	Mult.	N. Areas.	S. Areas.
1	N. 19° E.	27.00	25.53		8.79		2	1	25.51		8.80			4.33			
2	S. 77 E.	22.75		5.12	22.17		1	1		5.13	22.18		30.98		30.98 E.		158.9274
3	S. 27 E.	28.75		25.62	13.05		2	2		25.64	13.07		35.25		66.23 E.		1698.1372
4	S. 52 W.	14.50		8.93		11.42	1	1		8.94		11.41	1.66		67.89 E.		606.9366
5	S. 15½ E.	19.00		18.31	5.08		1	1		18.32	5.09			6.32	61.57 E.		1127.9624
6	West.	17.72				17.72	1	1		.01		17.71		12.62	48.95 E.		.4895
7	N. 36 W.	11.75	9.51			6.91	1	1	9.50			6.90		24.61	24.34 E.	231.2300	
8	North.	16.07	16.07				1	1	16.06		.01			6.89	17.45 E.	280.2470	
9	N. 62 W.	14.88	6.98			13.14	1	1	6.97			13.13		13.12	.4.33 E.	30.1801	
		172.42	58.09	57.98	49.09	49.19										541.6571	3592.4531
			57.98			49.09											541.6571
			.11 Error N.			.10 Error W.											2)3050.7960

Area 152 A. 2 R. 6.368 P.

152,5.398
4

2.1592
40

6.3680

EXAMPLE 5.

Sta.	Bearing.	Dist.	N.	S.	E.	W.	Cor. N.	Cor. W.	N.	S.	E.	W.	E. D. D.	W. D. D.	Mult.	N. Areas.	S. Areas.
1	S. 62° W.	7.57		3.56		6.68	1	1		3.55		6.69		11.46			
2	N. 43½ W.	5.89	4.28			4.05	1	1	4.29			4.06		10.75	10.75 W.		46.1175
3	North.	5.82	5.82				1	1	5.83			.01		4.07	14.82 W.		86.4006
4	N. 33½ W.	8.83	7.36			4.88	1	1	7.37			4.89		4.90	19.72 W.		145.3364
5	N. 48 E.	4.81	3.22		3.57		0	1	3.22		3.56			1.33	21.05 W.		67.7810
6	N. 12 E.	4.66	4.56		.97		0	0	4.56		.97		4.53		16.52 W.		75.3312
7	N. 62½ E.	5.27	2.43		4.68		1	1	2.44		4.67		5.64		10.88 W.		26.5472
8	S. 6½ E.	5.60		5.57	.64		1	1		5.56	.63		5.30		5.58 W.	31.0248	
9	S. 40½ E.	5.87		4.46	3.82		1	1		4.45	3.81		4.44		1.14 W.	5.0730	
10	East.	6.54			6.54		1	1	.01		6.53		10.34		9.20 E.	.0920	
11	North.	5.52	5.52				1	1	5.53			.01	6.52		15.72 E.	86.9316	
12	N. 68¼ E.	3.10	1.15		2.88		0	0	1.15		2.88		2.87		18.59 E.	21.3785	
13	S. 30 E.	7.90		6.84	3.95		1	1		6.83	3.94		6.82		25.41 E.		173.5503
14	S. 23 W.	8.80		8.10		3.44	1	1		8.09		3.45	.49		25.90 E.		209.5310
15	S. 31½ E.	6.42		5.48	3.35		1	1		5.47	3.34			.11	25.79 E.		141.0713
16	S. 50 W.	8.40		5.40		6.44	1	1		5.39		6.45		3.11	22.68 E.		122.2452
17	N. 44 W.	6.85	4.93			4.76	1	1	4.94			4.77		11.22	11.46 E.	56.6124	
			39.27	39.41	30.40	30.25										201.1123	1093.9117
				39.27	30.25												201.1123
			Error S.	14	15	Error E.											2)892.7994
																	44,6.3997
																	4
																	2.55988
																	40
																	22.39520

Area, 44 A. 2 R. 22.3952 P.

EXAMPLE 6.

Sta.	Bearing.	Dist.	N.	S.	E.	W.	E. D. D.	W. D. D.	Mult.	N. Areas.	S. Areas.
1	S. 85¼ E.	23.30		1.92	23.22		26.29				
2	S. 19 E.	31.12		29.42	10.13		33.35		33.35 E.		981.1570
3				(11.24)		(25.71)		15.58	17.77 E.		199.7348
4	N. 64 W.	29.72	13.03			26.71		52.42	34.65 W.		451.4895
5	N. 15½ W.	22.46	21.64			6.00		32.71	67.36 W.		1457.6704
6	N. 58 E.	25.94	13.75		22.00		16.00		51.36 W.		706.2000
7	S. 27¾ E.	6.60		5.84	3.07		25.07		26.29 W.	153.5336	
			48.42	48.42	58.42	58.42					3796.2517
											153.5336
											2)3642.7181
											182,1.35905
											4
											0.54362
											40
											21.7448

As diff. latitude 11.24 - - - - - - Ar. Co. 8.94923
Is to departure 25.71 - - - - - - - - - 1.41010
So is radius - - - - - - - - - - - - 10.00000

To tang. bearing 66° 23′ - - - - - - - - 10.35933

And,

As radius - - - - - - - - - - Ar. Co. 0.00000
Is to secant bearing 66° 23′ - - - - - - - 10.39727
So is diff. lat. 11.24 - - - - - - - - - - 1.05077

To distance 28.06 - - - - - - - - - - - 1.44804

EXAMPLE 7. (Pl. 2, fig. 4.)

Sta.	Bearing.	Dist.	N.	S.	E.	W.	Cor.	Cor.	N.	S.	E.	W.	E. D. D.	W. D. D.	Mult.	N. Areas.	S. Areas.
1	S. 60¾ W.	10.34		5.06		9.02				5.06		9.02		14.47			
2	N. 27¼ W.	17.88	15.89			8.18			15.89			8.18		17.20	17.20 W.		273.3080
3	N. 51 E.	15.85	9.97		12.32				9.97		12.32		4.14		13.06 W.		130.2082
4	N. (8¼) E.	9.61	(9.51)		(1.38)				9.51		1.38		13.70		.64 E.	6.0864	
5	S. (73°) E.	19.18		(5.61)	(18.34)					5.61	18.34		19.72		20.36 E.		114.2196
6	S. 16¾ E.	22.21		21.27	6.40		1	1		21.26	6.41		24.75		45.11 E.		959.0386
7	S. 71½ W.	16.66		5.29		15.80				5.29		15.80		9.39	35.72 E.		188.9588
8	N. 71¼ W.	5.76	1.85			5.45			1.85			5.45		21.25	14.47 E.	26.7695	
			37.22	37.23	38.44	38.45										32.8559	1665.7332
																	32.8559
																	2)1632.8773
																	81.6.43865
																	4
																	2.57546
																	40
																	23.0184

Area 81 A. 2 R. 23.0184 P.

As diff. lat. DF 3.91 - - - - - Ar. Co. 9.40782
Is to depart. 19.73 - - - - - - - - - - 1.29513
So is radius - - - - - - - - - - - - 0.00000

To tang. bearing N. 78° 47′ E. - - - - - - 10.70295

As radius - - - - - - - - Ar. Co. 0.00000
Is to secant bearing 78° 47′ - - - - - - - 10.71104
So is diff. latitude 3.91 - - - - - - - - - 0.59218

To distance DF 20.10 - - - - - - - - - 1.30322

FE 19.18
ED 9.61 - - - - - - - Ar. Co. 9.01728
DF 20.10 - - - - - - - - " " 8.69680

2)48.89

Half sum 24.445 - - - - - - - - - - 1.38819

Diff. 5.265 - - - - - - - - - - 0.72140

2)19.82367

Cos. ½ FDE 35° 17′ - - - - - - - 9.91183

FDE 70° 34′
Bearing DF N. 78 47 E.

Bearing DE N. 8° 13′ E.

As FE 19.18 - - - - - - - - - - - - 8.71715
Is to DE 9.61 - - - - - - - - - - - - 0.98272
So is sin. FDE 70° 34′ - - - - - - - - 9.97453

To sin. DFE - - 28° 12′ - - - - - - - 9.67440

Bearing FD S. 78 47 W.

106 59
180

Bearing EF S. 73 1 E.

EXAMPLE 8. (Fig. 81, *Surveying.*)

Sta.	Bearing.	Changed Bearing.	Dist.	N.	S.	E.	W.	E. D. D.	W. D. D.	Mult.	N. Areas.	S. Areas.
AB	N. 51¼ W.	North.		(26.47)					22.61			
BC	S. 45½ W.	N. 83¼ W.	15.16	1.78			15.06		15.06	15.06 W.		26.8068
CD	N. 50 W.	N. 1¼ E.	22.10	22.09		.48			14.58	29.64 W.		654.7476
DE	North.	N. 51¼ E.	18.83	11.79		14.69		15.17		14.47 W.		170.6013
EF	N. 48 E.	S. 80¾ E.	22.60		3.64	22.30		36.99		22.52 E.		81.9728
FG	N. 25½ W.	N. 25¾ E.	20.17	18.16		8.76		31.06		53.58 E.	973.0128	
GH	East.	S. 38¾ E.	26.57		20.72	16.63		25.39		78.97 E.		1636.2584
HI	S. 30½ E.	S. 20¾ W.	22.86		21.37		8.09	8.54		87.51 E.		1870.0887
IK	S. 44 W.	N. 84¾ W.	15.04	1.37			14.98		23.07	64.44 E.	88.2828	
KL	S. 47 E.	S. 4¼ W.	28.55		28.47		2.12		17.10	47.34 E.		1347.7698
LA	S. 20½ W.	S. 71¾ W.			(7.46)		(22.61)		24.73	22.61 E.		168.6706
				81.66	81.66	62.86	62.86				1061.2956	5956.9160

1061.2956

2)4895.6204

244.7.8102

4

3.12408

40

4.96320

Area 244 A. 3 R. 4.9632 P.

As radius	Ar. Co.	0.00000
Is to cosec. changed bearing LA 71° 45′		10.02241
So is departure 22.61		1.35430
To distance LA 23.81		1.37671

And,

As radius	Ar. Co.	0.00000
Is to cotang. bearing		9.51819
So is departure		1.35430
To diff. latitude 7.46		0.87249

EXAMPLE 9. (Fig. 80, *Surveying.*)

To find the third side.

Sta.	Bearing.	Dist.	N.	S.	E.	W.
EA	S. 52 W.	10.70		6.59		8.43
AB	S. 7½ W.	13.92		13.80		1.82
BC	S. 33¼ E.	9.00		7.53	4.93	
				27.92		10.25
						4.93
						5.32

As diff. lat. EC 27.92	Ar. Co.	8.55408
Is to depart. 5.32		0.72591
So is radius		10.00000
To tang. bearing S. 10° 47′ W.		9.27999

As radius	Ar. Co.	0.00000
Is to secant bearing 10° 47′		10.00774
So is diff. lat.		1.44592
To distance 28.42		1.45366

Sta.	Bearing.	Dist.	N.	S.	E.	W.	Cor. S.	Cor. W.	N.	S.	E.	W.	E. D. D.	W. D. D.	Mult.	N. Area.	S. Areas.
1	North.	7.81	7.81				2	1	7.79			.01	16.54				
2	S. 76¼ E.	18.15		4.32	17.63		3	3		4.35	17.60		17.59		17.59 E.		76.5165
3	S. 10¾ W.	28.42		27.92		5.30	5	5		27.97		5.35	12.25		29.84 E.		834.6248
4	N. 84¼ W.	27.12	2.72			26.98	5	4	2.67			27.02		32.37	2.53 W.		6.7551
5	N. 4½ W.	22.00	21.93			1.73	4	4	21.89			1.77		28.79	31.32 W.		685.5948
6	East.	16.58			16.58		3	3		.03	16.55		14.78		16.54 W.	.4962	
		120.08	32.46	32.24	34.21	34.01											1603.4912
																	.4962
																	2)1602.9950
																	80 1.4975
																	4
																	.59900
																	40
																	23.96000

Area, 80 A. 0 R. 23.96 P.

PROBLEM 12.

EXAMPLE 3.

Sta.	Bearing.	Dist.	N.	S.	E.	W.	Cor. S.	Cor. W.	N.	S.	E.	W.	E. D. D.	W. D. D.	Mult.	N. Areas.	S. Areas.
1	N. 41¼° E.	14.35	10.79		9.46		1		10.78		9.46		3.32				
2	S. 42¼ E.	14.71		10.89	9.89		1			10.90	9.89		19.35		19.35 E.		210.9150
3	S. 54 W.	16.32		9.59		13.20	1	1		9.60		13.21		3.32	16.03 E.		153.8880
4	N. 32¼ W.	11.50	9.72			6.14			9.72			6.14		19.35	3.32 W.		32.2704
			20.51	20.48	19.35	19.34											397.0734
															Double Area of Off-sets		61.3408
																	2)458.4142
																	22,9.2071
																	4
																	3.68284
																	40
																	27.31360

Dist.	Off-sets.	Int. Dist.	Sums.	Areas.
0.00	.38			
2.65	2.35	2.65	2.73	7.2345
3.80	1.70	1.15	4.05	4.6575
6.00	2.75	2.20	4.45	9.7900
7.50	1.40	1.50	4.15	6.2250
9.60	3.20	2.10	4.60	9.6600
12.38	2.72	2.78	5.92	16.4576
14.71	0.42	2.33	3.14	7.3162
				61.3408

Area, 22 A. 3 R. 27.3136 P.

EXAMPLE 4.

Sta.	Bearing.	Dist.	N.	S.	E.	W.	Cor. S.	Cor. E.	N.	S.	E.	W.	E. D. D.	W. D. D.	Mult.	N. Areas.	S. Areas.
1	S. 69½° E.	16.14		5.65	15.12		4	2		5.69	15.14		23.71				
2	S. 28 E.	9.38		8.29	4.41		2	1		8.31	4.42		19.56		19.56 E.		162.5436
3	S. 32¾ W.	21.20		17.83		11.47	6	2		17.89		11.45		7.03	12.53 E.		224.1617
4	N. 48 W.	22.47	15.03			16.70	6	2	14.97			16.68		28.13	15.60 W.		233.5320
5	N. 26¾ E.	19.00	16.97		8.55		5	2	16.92		8.57			8.11	23.71 W.		401.1732
		88.19	32.00	31.77	28.08	28.17											1021.4105
																	72.9743
																	37.8761
																	2)1132.2609
																	56 6.13045
																	4
																	2.45218
																	40
																	18.08720

0.00	.44			
3.80	2.00	3.80	2.44	9.2720
7.04	3.79	3.24	5.79	18.7596
9.87	2.34	2.83	6.13	17.3479
13.24	3.00	3.37	5.34	17.9958
16.14	0.31	2.90	3.31	9.5990
				72.9743

.00	.31			
2.67	2.94	2.67	3.25	8.6775
6.20	2.62	3.53	5.56	19.6268
9.38	0.39	3.18	3.01	9.5718
				37.8761

Area, 56 A. 2 R. 18.08720 P.

PROBLEM 13.

EXAMPLE 2. (Pl. 2, fig. 5.)

Here the various angles will be found to be as in the following proportions. Then,

To find log. of GA:

As sin. FAG 88° 30′	Ar. Co.	0.00015
Is to sin. GFA 68° 30′		9.96868
So is FG 20 ch.		1.30103
To GA		1.26986

To find log. GB:

As sin. FBG 42°	Ar. Co.	0.17449
Is to sin. GFB 24°		9.60931
So is FG		1.30103
To GB		1.08483

To find log. GC:

As sin. GCF 43° 15′	Ar. Co.	0.16419
Is to sin. GFC 38°		9.78934
So is FG		1.30103
To GC		1.25456

To find log. GD:

As sin. GDF 44° 30′	Ar. Co.	0.15434
Is to sin. GFD 59°		9.93307
So is GF		1.30103
To GD		1.38844

To find log. GE:

As sin. GEF 35° 30′	Ar. Co.	0.23605
Is to sin. GFE 103° 30′		9.98783
So is GF		1.30103
To GE		1.52491

To find 2 ABG:

As radius		Ar. Co.	0.00000
Is to sin. AGB	91°		9.99993
So is BG, AG	BG		1.08483
	AG		1.26986
To 2 ABG	226.268		2.35462

To find 2 BGC:

As radius		Ar Co.	0.00000
Is to sin. BGC	15° 15′		9.42001
So is GB, GC	GB		1.08483
	GC		1.25456
To 2 BGC	57.464		1.75940

To find 2 CGD:

As radius		Ar. Co.	0.00000
Is to sin. CGD	22° 15′		9.57824
So is GC, GD	GC		1.25456
	GD		1.38844
To 2 CGD	166.435		2.22124

To find 2 DGE:

As radius		Ar. Co.	0.00000
Is to sin. DGE	35° 30′		9.76395
So is GD, GE	GD		1.38844
	GE		1.52491
To 2 DGE	475.667		2.67730

To find 2 EGA:

As radius		Ar. Co.	0.00000
Is to sin. EGA	18°		9.48998
So is EG, GA	GE		1.52491
	GA		1.26986
To 2 EGA	192.641		2.28475

2 DGE	475.667
2 CGD	166.435
2 BGC	57.464
	892.207
2 AGB	226.268
	2)665.939
ABCDE	332.9695 Ch. = 33 A. 1 R. 7.512 P.

CHAPTER IV.

LAYING OUT AND DIVIDING LAND.

PROBLEM 1.

EXAMPLE 2.

Here, 325 Acres = 3250 chains.
And side = $\sqrt{3250} \doteq 57$ chains.

PROBLEM 2.

EXAMPLE 2.

Here breadth $= \frac{5 \text{ Acres}}{8 \text{ chains}} = \frac{50}{8} = 6.25$ chains.

PROBLEM 3.

EXAMPLE 2.

Here, 27 A. 3 R. 20 P. = 4460 P.
And, As 7 : 9 : : 4460 : 5734.2857.
$\sqrt{5734.2857} = 75.725$ = length.
Also, As 9 : 7 : : 75.725 : 58.897 = breadth.

PROBLEM 4.

EXAMPLE 2. (Pl. 2, fig. 6.)

Here, 114 A. 2 R. 33.4 P. = 1147.0875 chains.
Also, $\sqrt{1147.0875 + 7.55^2} = \sqrt{1204.09} = 34.7$.
And, 34.7 + 7.55 = 42.25 length.
34.7 — 7.55 = 27.15 breadth.

PROBLEM 5.

EXAMPLE 3. (Pl. 2, fig. 7.)

Here, 2 Acres = 320 Perches.

And,

As AB, sin. A	{ AB 30 P. -	Ar. Co.	8.52288
	{ sin. A 71° 15′	Ar. Co.	0.02368
Is to 2 ABC 640 - - - - - - - - - - -			2.80618
So is radius - - - - - - - - - - - -			10.00000
To AC 22.53 - - - - - - - - - - -			1.35274

EXAMPLE 4. (Pl. 2, fig. 8.)

As AB, sin. A	{ AB 32.26 -	Ar. Co.	8.49134
	{ sin. A 83° 30′	Ar. Co.	0.00280
Is to ABCD 740 - - - - - - - - - - -			2.86923
So is radius - - - - - - - - - - - -			10.00000
To AD 23.09 - - - - - - - - - - -			1.36337

PROBLEM 6.

EXAMPLE 2. (Pl. 2, fig. 9.)

Here, 27 A. 1R. 16 P. = 273.5 Ch.

And,

As ABC 273.5 - - - - - - - -		Ar. Co.	7.56304
Is to BDC 100 - - - - - - - - - - -			2.00000
So is AB 35.20 - - - - - - - - - - -			1.54654
To BD 12.87 - - - - - - - - - - -			1.10958

PROBLEM 7.

EXAMPLE 2. (Pl. 2, fig. 10.)

Construction.

Make AB, equal to the greater of the given sides (20). Draw BD perpendicular to AB, equal to twice the given area, divided by AB (12.39). Through D draw DC parallel to AB. Then if AC be made equal to the other given side (16.25), and BC be joined; ABC will be the triangle.

For the Division Line. Make AP = 8.50 the given distance. Take AF to AC in the ratio of the part to be cut off to the whole area. Join PF, draw BG parallel to it; then PG will be the division line.

Demonstration.

AB : AP : : AG : AF, Therefore, AB. AC : AP. AG:: AC. AG : AG. AF : : AC : AF, or AB. AC. sin. A : AP. AG. sin. A : : AC : AF : : m : n (m being the whole area, and n the part to be cut off.) Hence, since AC. AB sin. A = m, AP. AG sin. A = n, and PG is the division line.

Calculation.

As ABC 123.9375		Ar. Co.	7.90680
Is to APG 30			1.47712
So is AB. AC	AB 20		1.30103
	AC 16.25		1.21085
To AP. AG			1.89580
AP = 8.50			0.92942
AG = 9.255			0.96638

PROBLEM 8.

EXAMPLE 2. (Pl. 2, fig. 11.)

Here, As BAC 100 ch.		Ar. Co.	8.00000
Is to BDG 45			1.65321
So is BA^2	BA 25		1.39794
	BA		1.39794
To BD^2			2)2.44909
BD = 16.77			1.22454

PROBLEM 9.

EXAMPLE 2. (Pl. 2, fig. 12.)

Here the angles are, A = 71° 45′, B = 49° 15′, and C = 59°. Hence,

As sin. A. sin. B	sin. A	71° 45′	Ar. Co.	0.02241
	sin. B	49° 15′	" "	0.12058
Is to rad.. sin. C	radius			10.00000
	sin. C	59°		9.93307
So is 2 ABC 80 ch.				1.90309
To AB^2				2)1.97915
AB = 9.763				.98957

PROBLEM 10.

EXAMPLE 2. (Pl. 2, fig. 13.)

Here the angles A = 99° 30′, B = 122°, and P = 41° 30′.

And, As sin. A. sin. B	sin. A 99° 30′	Ar. Co.		0.00600
	sin. B 122°	"	"	0.07158
Is to rad. . sin. P	radius			10.00000
	sin. P 41° 30′			9.82126
So is 2 ABCD 50 ch.				1.69897
To fourth term 39.61				1.59781

AB^2 - - 36

CD^2 - $\sqrt{75.61} = 8.695$.

Also, As sin. P 41° 30′	Ar. Co.		0.17874
Is to sin. B 122°			9.92842
So is DC—AB 2.695			0.43056
To AD 3.449			0.53772

EXAMPLE 3. (Pl. 3, fig. 1.)

Here the angles are, A = 90°, B = 73° 30′, and P = 16° 30′.

Also, As sin. A. sin. B	sin. A 90°	Ar. Co.		0.00000
	sin. B 73° 30′	"	"	0.01826
Is to rad. . sin. P	radius			10.00000
	sin. P 16° 30′			9.45334
So is 2 ABCD 160 ch.				2.20412
To fourth term 47.39				1.67572

AB^2 - 182.25

$CD = \sqrt{134.86} = 11.61$.

And, As sin. P 16° 30′	Ar. Co.		0.54666
Is to sin. B 73° 30′			9.98174
So is AB—CD 1.89			0.27646
To AD 6.38			0.80486

PROBLEM 11.

EXAMPLE 2. (Pl. 3, fig. 2.)

Here the angles are, A = 90°, B = 73° 30′, C = 72° 45′, D = 123° 45′, and P = 16° 30′.

As sin. C . sin. D	sin. C 72° 45′	Ar. Co.	0.01999
	sin. D 123° 45′	" "	0.08015
rad., sin. P	rad. - - - - - - -		10.00000
	sin. P 16° 30′ - - -		9.45334
So is 2 ABCD 160 ch. - - - - - - - -			2.20412
To fourth term 57.23 - - - - - - - - -			1.75760

And, As sin. C . sin. D	sin. C - - -	Ar. Co.	0.01999
	sin. D - - -	" "	0.08015
Is to sin. A . sin. B	sin. A 90° - - - -		10.00000
	sin. B 73° 30′ - - -		9.98174
So is AB^2	AB 13.50 - - - -		1.13033
	AB - - - - - - - -		1.13033
To fourth term 220.06 - - - - - - - -			2.34254
57.23			

$$CD = \sqrt{162.83} = 12.76$$

As sin. C 72° 45′ - - - - - - -	Ar. Co.	0.01999
Is to sin. B 73° 30′ - - - - - - - - -		9.98174
So is AB 13.50 - - - - - - - - - -		1.13033
To CS 13.554 - - - - - - - - - - -		1.13206
CD 12.76		
SD = .794		

As sin. P 16° 30′ - - - - - -	Ar. Co.	0.54666
Is to sin. C 72° 45′ - - - - - - - - -		9.98001
So is SD .794 - - - - - - - - - -		1.89982
To AD 2.67 - - - - - - - - - - -		0.42649

EXAMPLE 3. (Pl. 3, fig. 3.)

Here the angles are, B = 133°, A = 68° 15′, C = 64°, D = 94° 45′, and P = 21° 15′.

As sin. C. sin. D	sin. C 64° -	Ar. Co.	0.04634
	sin. D 94° 45′	" "	0.00149
Is to rad. sin. P	radius - - - - - - -		10.00000
	sin. P 21° 15′ - - -		9.55923
So is 2 ABCD 400 ch. - - - - - - - -			2.60206
To fourth term 161.85 - - - - - - - -			2.20912

As sin. C, sin. D	sin. C - - -	Ar. Co.	0.04634
	sin. D - - -	" "	0.00149
Is to sin. A, sin. B	sin. A 68° 15′ - - - -		9.96793
	sin. B 133° - - - - -		9.86413
So is AB^2	AB 17.24 - - - -		1.23654
	AB - - - - - - -		1.23654
To fourth term 225.41 - - - - - - - -			2.35297
161.85			

CD = $\sqrt{387.26}$ = 19.68.

As sin. C 64° - - - - - - - -		Ar. Co.	0.04634
Is to sin. B 133° - - - - - - - - - -			9.86413
So is AB 17.24 - - - - - - - - - -			1.23654
To CS 14.03 - - - - - - - - - - -			1.14701
CD 19.68			
SD = 5.65			

As sin. P 21° 15′ - - - - - - -		Ar. Co.	0.44077
Is to sin. C 64° - - - - - - - - - -			9.95366
So is SD 5.65 - - - - - - - - - -			0.75205
To AD 14.01 - - - - - - - - - -			1.14648

PROBLEM 12.

EXAMPLE 2. (Pl. 3, fig. 4.)

To find bearing and distance of AB and area of AKB.

Sta.	Bearing.	Dist.	N.	S.	E.	W.	E. D. D.	W. D. D.	Mult.	S. Areas.	N. Areas.
AK	N. 19 E.	18.00	17.02		5.86			14.86			
KB	S. 77 E.	15.25		3.43	14.86		20.72		20.72 E.	71.0686	
AB				13.59		20.72		5.86	14.86 E.	201.9474	
										273.0170	
										700.0000	
										426.9830	

Twice the given area = 2 ABCD =

As diff. lat. AB 13.59 - - - - - - Ar. Co. 8.86678
Is to departure 20.72 - - - - - - - - - 1.31639
So is radius - - - - - - - - - - - - 10.00000
To tang. bearing N. 56° 44′ E. - - - - - 10.18317

As radius - - - - - - - - - - -	Ar. Co.	0.00000
Is to secant of bearing 56° 44′ - - - - -		10.26079
So is diff. lat. 13.59 - - - - - - - - -		1.13322
To distance AB 24.78 - - - - - - - -		1.39401

The angles will now be found to be as follow, viz.:
DAB = 61° 1′, ABC = 83° 44′, BCD = 70° 30′,
CDA = 144° 45′, and P = 35° 15′.

As sin. C sin. D	sin. C 70° 30′	Ar. Co.	0.02565
	sin. D 144° 45′	Ar. Co.	0.23871
Is to rad. sin. P	rad. - - - - - - -		10.00000
	sin. P 35° 15′ - - -		9.76129
So is 2 ABCD 426.983 - - - - - - - -			2.63041
To Fourth term 452.96 - - - - - - -			2.65606

And,

As sin. C. sin. D	sin. C - - -	Ar. Co.	0.02565
	sin. D - - -	Ar. Co.	0.23871
Is to sin. A. sin. B	sin. A 61° 1′ - - -		9.94189
	sin. B 83° 44′ - - -		9.99740
So is AB^2	AB 24.78 - - - -		1.39401
	AB - - - - - - -		1.39401
To Fourth term 981. - - - - - - -			2.99167

$$CD = \sqrt{\overline{452.96}\ 528.04} = 22.98$$

As sin. C 70° 30′ - - - - - - - -		Ar. Co.	0.02565
Is to sin. B 83° 44′ - - - - - - - - -			9.99740
So is AB 24.78 - - - - - - - - - -			1.39401
To CS 26.13 - - - - - - - - - -			1.41706
CD 22.98			
SD 3.15			

As sin. P 35° 15′ - - - - - - - -			0.23871
Is to sin. C 70° 30′ - - - - - - - -			9.97435
So is SD 3.15 - - - - - - - - - -			0.49831
To AD 5.145 - - - - - - - - - - -			0.71137

PROBLEM 13.

EXAMPLE 2. (Pl. 3, fig. 5.)

Sta.	Bearings.	Changed Bearings.	Dist.	N.	S.	E.	W.	E. D. D.	W. D. D.	Mult.	N. Areas.	S. Areas.
AB	S. 78 W.	N. 78¾ W.	8.00	1.56			7.85		17.76	17.76 W.		27.7056
BC	N. 26½ W.	N. 3¼ W.	11.08	11.06			.62		8.47	26.23 W.		290.1038
CD	N. 38½ E.	N. 61¾ E.	12.82	6.07		11.29		10.67		15.56 W.		94.4492
DE	S. 64 E.	S. 40¾ E.	10.86		8.23	7.09		18.38		2.82 E.		23.2086
EI	S. 23¼ E.	South.			(6.51)			7.09		9.91 E.		
IA					(3.95)		(9.91)		9.91	0.00		
				18.69	18.69	18.38	18.38					435.4672

Twice given Area, 500.

9.91)64.5328(6.51
59 46
5072
4955
1178
991
187

As diff. latitude IA 3.95 - - - - Ar. Co. 9.40340
Is to departure 9.91 - - - - - - - - - 0.99607
So is radius - - - - - - - - - - - 10.00000

To tang. bearing 68° 16′ - - - - - - - 10.39947
23 15
45° 1′

As radius - - - - - - - - - - Ar. Co. 0.00000
Is to secant bearing 68° 16′ - - - - - - - 10.43146
So is diff. lat. 3.95 - - - - - - - - - - 0.59660

To distance AI 10.67 - - - - - - - - - 1.02806

PROBLEM 14.

EXAMPLE 2. (Pl. 3, fig. 6.)

Here, As $2 : 1 :: BC^2(100) : EF^2 = 50$,

$EF = \sqrt{50} = 7.07.$

And, As BC (10) : EF (7.07) :: AB (15) : AF = 10.605.

PROBLEM 15.

EXAMPLE 2. (Pl. 3, fig. 7.)

Here the angles are, A = 36° 30′, B = 100° 30′, C = 43°, E = 74° 30′, and F = 69°.

As sin. E. sin. F	sin. E 74° 30	Ar. Co.	0.01609
	sin. F 69 - -	" "	0.02985
Is to sin. C. sin. B	sin. C 43° - - - - -		9.83378
	sin. B 100° 30′ - - -		9.99267
So is BC^2	BC 18.66 - - - - -		1.27091
	BC - - - - - - -		1.27091
To fourth term	259.54 - - - - - - - - -		2.41421

$$\begin{array}{r} 4 \\ \hline 9)1038.16 \end{array}$$

$EF = \sqrt{115.35} = 10.74.$

As sin. A	36° 30′ - - - - - -	Ar. Co.	0.22561
Is to sin. E	74° 30′ - - - - - - - -		9.98391
So is EF	10.74 - - - - - - - - -		1.03100
To AF	17.40 - - - - - - - - -		1.24052

PROBLEM 16.

EXAMPLE 2. (Pl. 3, fig. 8.)

Here, $EF = \sqrt{\left(\frac{AB^2 + CD^2}{2}\right)} = \sqrt{5796.18} = 76.13.$

And, DC—AB (29.4) : FE—AB (16.13) :: AD (30) : AF = 16.46.

PROBLEM 17.

EXAMPLE 2. (Pl. 3, fig. 9.)

Here, the angles are A = 80° 30′, B = 123° 30′, C = 90°, D = 66°, and P = 24°.

As sin. A . sin. B	sin. A 80° 30′	Ar. Co.	0.00600
	sin. B 123° 30′	" "	0.07889
Is to sin. C . sin. D	sin. C 90° - - - - -		10.00000
	sin. D 66° - - - - -		9.96073
So is CD^2	CD 13.33 - - - - -		1.12483
	CD - - - - - - -		1.12483
To Fourth term	197.37 - - - - -		2.29528

3

592.11

$4\ AB^2$ = 207.36

7)799.47

EF = $\sqrt{114.21}$ = 10.69

As sin. P 24° - - - - - - - - -	Ar. Co.	0.39069
Is to sin. E 123° 30′ - - - - - - - - -		9.92111
So is FE—AB 3.49 - - - - - - - - -		0.54283
To AF 7.155 - - - - - - - - - - -		0.85463

EXAMPLE 3. (Pl. 3, fig. 10.)

Here the angles are as in the last example.

As sin. C . sin. D	sin. C 90° -	Ar. Co.	0.00000
	sin. D 66° -	" "	0.03927
Is to sin. A . sin. B	sin. A 80° 30′ - - -		9.99400
	sin. B 123° 30′ - -		9.92111
So is AB^2	AB 7.20 - - - - -		0.85733
	AB - - - - - - -		0.85733
To Fourth term	46.67 - - - -		1.[illegible]04

4

186.68

$3\ CD^2$ 533.0667

7)719.7467

EF = $\sqrt{102.8209}$ = 10.14

As sin. P 24° - - - - - - - - -	Ar. Co.	0.39069
Is to sin. E 90° - - - - - - - - - -		10.00000
So is CD—EF 3.19 - - - - - - - - -		0.50379
To DF 7.843 - - - - - - - - -		0.89448

AD 18

AF = 10.157

PROBLEM 18.

EXAMPLE 2. (Pl. 3, fig. 11.)

Here the angles A, B, C, D, and P, are as in the last Problem, also E = 67° and F = 89°.

As sin. E . sin. F	sin. E 67° -	Ar. Co.	0.03597
	sin. F 89° -	" "	0.00007
Is to sin. A . sin. B	sin. A 80° 30' - - -		9.99400
	sin. B 123° 30' - -		9.92111
So is AB^2	AB 7.20 - - - - -		0.85733
	AB - - - - - -		0.85733
To Fourth term	46.32 - - - -		1.66581
	4		
	185.28		

As sin. E . sin. F	sin. E 67° -	Ar. Co.	0.03597
	sin. F 89° -	" "	0.00007
Is to sin. C . sin. D	sin. C 90° - - - -		10.00000
	sin. D 66° - - - -		9.96073
So is CD^2	CD 13.33 - - - -		1.12483
	CD - - - - - - -		1.12483
Fourth term	176.37 - - - -		2.24643
	3		
	529.11		
	185.28		
	7)714.39		

EF = $\sqrt{102.0557}$ = 10.10

As sin. E 67° - - - - - - - - Ar. Co. 0.03597
Is to sin. B 123° 30′ - - - - - - - - - 9.92111
So is AB 7.20 - - - - - - - - - - - 0.85733

To ES - 6.52 - - - - - - - - - - 0.81441
EF - 10.10

FS - 3.58

As sin. P. 24° - - - - - - - - Ar. Co. 0.39069
Is to sin. E 67° - - - - - - - - - - 9.96403
So is FS 3.58 - - - - - - - - - - 0.55388

To AF 8.102 - - - - - - - - - - 0.90860

PROBLEM 19.

EXAMPLE 3. (Pl. 3, fig. 12.)

The area of this tract has been found (Ex. 3, Prob. 11, Chap. III.) to be 858.552 square chains. (The latitudes and departures used in the subsequent operations are taken from that example.)

To find area ABCDE, and the latitude and departure of EA.

	N.	S.	E.	W.	E. D. D.	W. D. D.	Mult.	N. Area.	S. Areas.
AB		9.15		6.46		1.29			
BC	17.21			17.20		23.66	23.66 W.		407.1886
CD	10.41		2.89			14.31	37.97 W.		395.2677
DE		3.61	15.60		18.49		19.48 W.	70.3228	
EA		(14.86)	(5.17)		20.77		1.29 E.		19.1694
	27.62	27.62	23.66	23.66					821.6257
									70.3228
									751.3029
								2 ABCDEI	858.552
									2)107.2491
								AEI	53.6245

	N.	S.	E.	W.	E. D. D.	W. D. D.	Mult.	N. Area.	S. Area.
AE	14.86			5.17		21.49			
EF	.93		21.49		16.32		16.32 E.	15.1776	
FA		(15.79)		(16.32)	5.17		21.49 E.		339.3271
									15.1776
									2)324.1495
								AEF	162.07475

As AEF 162.07475 - - - - - - Ar. Co. 7.79028
Is to AEI 53.6245 - - - - - - - - - - 1.72937
So is lat. EF .93 - - - - - - - - - —1.96848

To lat. EI .31 - - - - - - - - - - - —1.48813

As AEF - - - - - - - - - - Ar. Co. 7.79028
Is to AEI - - - - - - - - - - - - 1.72937
So is depart. EF 21.49 - - - - - - - - 1.33224

To depart. EI 7.11 - - - - - - - - - 0.85189

	N.	S.	E.	W.
AE	14.86			5.17
EI	.31		7.11	
IA		(15.17)		(1.94)

As diff. lat. 15.17 - - - - - - - Ar. Co. 8.81901
Is to depart. 1.94 - - - - - - - - - 0.28780
So is radius - - - - - - - - - - - - 10.00000

To tang. bearing AI 7° 17′ - - - - - - - 9.10681

As radius - - - - - - - - - - Ar. Co. 0.00000
Is to secant bearing - - - - - - - - - 10.00352
So is diff. lat. 15.17 - - - - - - - - - 1.18099

To dist. AI 15.29 - - - - - - - - - - 1.18451

CHAPTER VI.

MISCELLANEOUS QUESTIONS.

QUESTION 1.

Here $\frac{1}{4}$ Acre = 2420 square yards;

And radius $= \sqrt{\left(\frac{2420}{3.1416}\right)} = \sqrt{770.3081} = 27.75.$

QUESTION 2.

Construction.

Make AB (Pl. 4, fig. 1) = 40 = one of the given sides, and at A draw AL perpendicular to AB and $= \frac{320}{40} = 8$; through L draw GH parallel to AB, and with the centre A and distance = 20 = the other given side, describe an arc, cutting GH in D and C; join AC, BC, AD, and BD: then will ABC and ABD answer the conditions of the question.

Calculation.

$AE = AF = \sqrt{AC^2 - CE^2} = \sqrt{400 - 64} = 18.3303$; and BE = AB—AE = 21.6697; therefore, $BC = \sqrt{BE^2 + EC^2} = \sqrt{533.57589809} = 23.099$. Also, BF = AB+AF=58.3303, and $BD = \sqrt{BF^2 + FD^2} = \sqrt{3466.42389809} = 58.876$.

Another Solution.

Find AE = AF as before. Then (Euclid, Book II. 13) $BC^2 = AB^2 + AC^2 - 2\,AB.AE = 2000 - 1466.424 = 533.576$, and BC = 23.099. Also (Euclid, II. 12), $BD^2 = AB^2 + AD^2 + 2\,AB.AF = 2000 + 1466.424 = 3466.424$, and BF=58.876.

QUESTION 3.

Here it is evident the number of acres will be inversely as the number of square yards in a Perch:

Therefore, $6^2 : 5.5^2 :: 110$ A. : 92 A. 1 R. $28\frac{4}{9}$ P. Cheshire.

And $7^2 : 5.5^2 :: 110$ A. : 67 A. 3 R. $25\frac{1}{49}$ P. Irish.

QUESTION 4.

Here $\frac{28}{12} = \frac{7}{3}$ = twice the thickness of the wall, also 840 links = 554.4 feet = the longer diameter within the walls; 612 links = 403.92 feet = the shorter; $554.4 + \frac{7}{3} = \frac{1670.2}{3}$ = longer diameter outside, and $403.92 + \frac{7}{3} = \frac{1218.76}{3}$ = shorter. By Prob. 10, Chap. III. the area within the walls = $554.4 \times 403.92 \times .7854 = 223933.248 \times .7854 =$ 175877.1729792 ft. = 4 A. 0 R. 6 P. The area to the outside = $\frac{1670.2}{3} \times \frac{1218.76}{3} \times .7854 = \frac{2035572.952}{9} \times .7854 = \frac{1598738.9965008}{9}$ = 177637.6662778 feet. Therefore 177637.666 — 175877.173 = 1760.493 = area the wall stands upon.

QUESTION 5.

Here the area of an ellipse whose diameters are 3 and 2 is 4.7124. Then, since similar figures are as the squares of their like dimensions, we have, As 4.7124 : 160 :: 9 : 305.5768 = square of the longer diameter; consequently $\sqrt{305.5768} = 17.481$ = longer diameter; and 3 : 2 :: 17.481 : 11.654 = shorter diameter.

QUESTION 6.

Find the area of the triangle whose sides are 9, 8, and 6; thus, $\frac{9+8+6}{2} = 11.5$, and $\sqrt{11.5 \times 2.5 \times 3.5 \times 5.5} = \sqrt{553.4375} = 23.525$ square perches. Also, 6 A. 1 R. 12 P. = 1012 P., and 23.525 : 1012 :: 8^2 : 2753.1562 = square of the second side; therefore $\sqrt{2753.1562}$ = 52.47. Also,

8 : 9 :: 52.47 : 59.029 = longest side.

8 : 6 :: 52.47 : 39.353 = shortest side.

QUESTION 7. (Pl. 4, fig. 2.)

To find ABC:

AB	27.35	
BC	31.15	
CA	38.00	
	2)96.50	
Half sum	48.25	1.68350
Remainders	20.90	1.32015
	17.10	1.23300
	10.25	1.01072
		2)5.24737
ABC 420.418		2.62368

To find ACE:

AC	38.	
CE	40.10	
EA	22.20	
	2)100.30	
Half sum	50.15	1.70027
Remainders	12.15	1.08458
	10.05	1.00217
	27.95	1.44638
		2)5.23340
ACE 413.71		2.61670

To find CED:

CE	40.10	
CD	23.70	
DE	29.25	
	2)93.05	
Half sum	46.525	1.66768
Remainders	6.425	0.80787
	22.825	1.35841
	17.275	1.23741
		2)5.07137
CED 343.308		2.53568

Hence the whole area = 420.418+413.71+343.308 = 1177.436 Ch. = 117 A. 2 R. 38.976 P.

QUESTION 8.

Construction.

Make AB (Pl. 4, fig. 3.) equal to half the given perimeter = 52, and bisect it in D; make DC perpendicular to AB and equal to the square root of the given area; with the centre C and radius equal to AD, describe an arc cutting AB in E, complete the rectangle AEFG and it will be the one required. The demonstration is evident from (Euclid XXVIII. 6.)

Calculation.

$DE = \sqrt{CE^2 - CD^2} = \sqrt{676 - 480} = \sqrt{196} = 14.$

$AE = AD + DE = 26 + 14 = 40$, and $EF = EB = 26 - 14 = 12$.

QUESTION 9.

Construction.

Draw any line AC, (Pl. 4, fig. 4.) and in it take AE = 20 = given difference; make EF perpendicular to AC = 20; join AF and produce it to B, making FB = 20; then will AB be a side of the square.

Demonstration.

Since EA = EF, the angles FAE and AFE are each equal to half a right angle, and AC must be the diagonal of the square. Again the triangles CEF and CBF are equal, since they are right angled at E and B, and have the hypothenuse and one leg in each equal: we have therefore CE = CB = CA—20.

Calculation.

$AF = \sqrt{AE^2 + EF^2} = \sqrt{800} = 28.284$, and $AB = AF + FB = 48.284$; hence the area = AB^2 = 2331.344656 sq. per. = 14 A. 2 R. 11.34 P.

QUESTION 10.

Construction.

Let ABCD (Pl. 4, fig. 5.) be the given rectangle. In BA and BA produced take BH = BC, and AR = ¾ AD. On BR describe the semicircle BPR, meeting DA produced in P; bisect AH in O, and with the centre O and radius OP, describe the semicircle EPQ, make AG = AQ, complete the rectangle AF, and the thing is done.

Demonstration.

$AF = AE \times AG = AE \times AQ = AP^2 = AB \times AR = \frac{3}{4} AB \times AD = \frac{3}{4} AC$. Also, $BE = BH - HE = BC - AQ = AD - AG = GD$.

Calculation.

$AO = \frac{1}{2} AH = 10$: $AP^2 = AB \times \frac{3}{4} AD = 6000$; therefore, $OP = \sqrt{AP^2 + OA^2} = \sqrt{6100} = 78.1025$; $BE = BO - OE = 90 - 78.1025 = 11.8975$.

QUESTION 11.

Construction.

Let ABD (Pl. 4, fig. 6.) be the given circle. Draw the diameter AB and radius CD perpendicular to it; take $CF = \frac{1}{5} AC$; upon BF describe a semicircle cutting CD in E: with C as a centre and radius CE, describe the circle EGH, and the thing is done.

Demonstration.

CE is a mean proportional between CF and CB; hence $CF : CB :: CE^2 : CB^2 :: 4 : 5$; and since circles are as the squares of their radii, we have $GEH = \frac{4}{5} ABD$.

Calculation.

$$\sqrt{5} : \sqrt{4} :: AC\ (75) : EC = \frac{75\sqrt{4}}{\sqrt{5}}$$

$$= \frac{150\sqrt{5}}{5} = 30\sqrt{5} = 67.082, \text{ and } DE = DC - EC = 7.918.$$

QUESTION 12.

Construction.

With the given distances form the triangle ABC, (Pl. 4, fig. 7.) Upon AB describe the equilateral triangle ABD; join CD and on it describe the equilateral triangle CDE, which will be the one required.

Demonstration.

Since BD and BC are by construction two of the given distances; it is only necessary to prove that $BE = AC$, which is evident from the equality of the triangles DAC and DBE.

Calculation.

In the triangle ABC, find the angle BAC, thus,

BC	10		
AC	7.5	Ar. Co.	9.12494
AB	12.5	Ar. Co.	8.90309
	2)30.		
	15		1.17609
	5		0.69897
			2)19.90309
Cos. ½ BAC	26° 34′		9.95154
BAC	53° 8′		

Then in the triangle DAC we have DA and AC, and the angle DAC = 113° 8′ to find DC, thus,

As DA+AC	20	Ar. Co.	8.69897
Is to DA—AC	5		0.69897
So is tang. $\frac{DCA+ADC}{2}$	33° 26′		9.81968
To tang. $\frac{DCA—ADC}{2}$	9° 22′		9.21762
ACD	42° 48′		

And,

As sin. ACD	42° 48′	Ar. Co.	0.16785
Is to sin. DAC	113° 8′		9.96360
So is AD	12.5		1.09691
To DC	16.92		1.22836

Then in CDE, we have the sides and angles to find the area, thus,

As radius		Ar. Co.	0.00000
Is to sin CDE	60°		9.93753
So is CD×DE	CD		1.22836
	DE		1.22836
To 2 CDE	247.88		2.39425

123.94 Ch. = 12 A. 1 R. 23.04 P.

QUESTION 13.

Construction.

With the given bearings and distances protract the figure ABCDfg, Pl. 4, fig. 8. Join Ag, and with the centres g and A, and distances equal to the 4th and 7th sides, describe arcs cutting in G. Join AG and gG, and draw DE, EF, and FG respectively parallel and equal to gG, Df, and fg. Then will ABCDEFG be the required map.

Calculation.

To find the bearing and distance of gA.

	Bearing.	Dist.	N.	S.	E.	W.
AB	S. 72 W.	24.00		7.42		22.83
BC	North.	38.00	38.00			
CD	N. 82½ E.	41.00	5.35		40.65	
Df	S. 80 E.	11.50		2.00	11.32	
fg	S. 26 W.	22.00		19.77		9.64
gA				(14.16)		(19.50)
			43.35	43.35	51.97	51.97

As diff. lat. 14.16 - - - - - - - - Ar. Co. 8.84894
Is to departure 19.50 - - - - - - - - - 1.29003
So is radius - - - - - - - - - - - - 10.00000
To tang. bearing gA S. 54° 1′ W. - - - - 10.13897

As radius - - - - - - - - - Ar. Co. 0.00000
Is to secant bearing 54° 1′ - - - - - - - 10.23096
So is diff. lat. 14.16 - - - - - - - - - 1.15106
To distance gA 24.10 - - - - - - - - 1.38202

In the triangle AGg we have the sides to find the angles AgG and GAg;

Thus,

AG	37		
gG	20	Ar. Co.	8.69897
Ag	24.1	Ar. Co.	8.61798
	2)81.1		
Half sum	40.55	- - - - -	1.60799
Remainder	3.55	- - - - -	0.55023
			2)19.47517
Cos. ½ AgG	56° 52½′	- - - -	9.73758
AgG	113° 45′		

And,

As AG 37	- - - - - - - - - -	Ar. Co.	8.43180
Is to gG 20	- - - - - - - - - - - - -		1.30103
So is sin. AgG	113° 45′ - - -	- - - - -	9.96157
To sin. gAG	29° 39′ - - - - - - - - - -		9.69440

Applying now the bearing of gA to these angles we will have the bearing of gG or DE = S. 59° 44′ E, and of GA = S. 83° 40′ W. The area will then be calculated as in the following table, viz.

Sta.	Bearing.	Dist.	N.	S.	E.	W.	Cor. S.	Cor. W.	N.	S.	E.	W.	E. D. D.	W. D. D.	Mult.	N. Areas.	S. Areas.
AB	S. 72 W.	24.00		7.42		22.83	1			7.43		22.83		59.61			
BC	North.	38.00	38.00				1		37.99					22.83	22.83 W.		867.3117
CD	N. 82½ E.	41.00	5.35		40.65		1		5.34		40.65		40.65		17.82 E.	95.1588	
DE	S. 59¾ E.	20.00		10.08	17.28					10.08	17.28		57.93		75.75 E.		763.5600
EF	S. 80 E.	11.50		2.00	11.32					2.00	11.32		28.60		104.35 E.		208.7000
FG	S. 26 W.	22.00		19.77		9.64	1			19.78		9.64	1.68		106.03 E.		2097.2734
GA	S. 83¾ W.	37.00		4.03		36.78	1			4.04		36.78		46.42	59.61 E.		240.8244
			43.35	43.30	69.25	69.25											4177.6695
																	95.1588
																	2)4082.5107
																	4(0)204.1.2553
																	4)51.1.25 P.
																	12 A. 3 R. 1.25 P.

QUESTION 14.

Construction.

Make AB, (Pl. 4, fig. 9.) = the given side, and divide it in D, so that AD may be to DB in the ratio of 3 to 2; in AB produced, take DO a fourth proportional to AD—DB, DB, and AD, and with the centre O and radius OD, describe the semicircle DCE; make AG perpendicular to AB, and equal to twice the area divided by AB = 6; through G draw GF parallel to AB, cutting the circle in C and F; join AC BC, AF and BF; then will ABC and ABF answer the conditions of the question.

Demonstration.

Since AD—DB : DB : : AD : DO, we have AD : DB : : AO : DO or AO : AD : : DO : DB, therefore, AO : DO : : DO : OB, consequently (Euclid, F. 6.) AC : BC : : AD : DB : : 3 : 2; and AF : BF : AD : DB : : 3 : 2.

Calculation.

As 3+2 : 15 : : 3 : AD = 9, and DB = 6; also, 9—6 : 6 : : 9 : DO = 18, and AO = 9+18 = 27, join OC, and OF, and let fall the perpendiculars CL and FP; then OL = $\sqrt{OC^2—CL^2}$ = $\sqrt{324—36}$ = $\sqrt{288}$ = 16.9706, and AL = AO—OL = 10.0294; hence AC = $\sqrt{AL^2+LC^2}$ = $\sqrt{136.58886436}$ = 11.6871; and as 3 : 2 : : 11.6871 : BC = 7.7914. Again AP = AO+OP = 43.9706, and AF = $\sqrt{AP^2+PF^2}$ = $\sqrt{1969.41366436}$ = 44.3781; and as 3 : 2 : : 44.3781 : BF = 29.5854

QUESTION 15.

Construction.

Make AB, (Pl. 4, fig. 10.) = the given side, and BL = the sum of the other sides; Bisect AB in D, and take DH a third proportional to 2 AB and BL; Draw HE perpendicular to BH and equal to $\frac{1600}{50}$ = 32. Through E draw EF parallel and equal to BL; join EA and produce it to G, making FG = AB; draw AC parallel to FG, and join BC; then ABC is the triangle required.

Demonstration.

By Construction BL^2 = 2 AB × DH; also, in the similar triangles EGF and EAC, we have GF (AB) : AC : : EF (BL) : EC (HP).

Hence $BL \times AC = GF \times HP$, or $2BL \times AC = 2GF \times HP$. Subtracting these equals from the preceding, we have $BL^2 - 2BL \times AC = 2AB \times DH - 2GF \times HP = 2AB \times DP = (BP + AP) \times (BP - AP) = BP^2 - AP^2 = BC^2 - AC^2$. Hence $BL^2 - 2BL \times AC + AC^2 = BC^2$, and $BL - AC = BC$, or $BL = BC + AC$.

Calculation.

As 2 AB (100) : BL (85) :: BL (85) : DH = 72.25, and AH = DH — AD = 47.25. Now in the right angled triangle AHE, we have the sides AH and HE, to find HAE and AE; thus,

As AH 47.25 - - - - - - - -	Ar. Co.	8.32560
Is to HE 32 - - - - - - - - - -		1.50515
So is radius - - - - - - - - - -		10.00000
To tang. HAE 34° 6½′ - - - - - - - -		9.83075

And,

As radius - - - - - - - - - -	Ar. Co.	0.00000
Is to sec. HAE 34° 6½′ - - - - - - - -		10.08198
So is AH - - - - - - - - - - - -		1.67440
To AE 57.07 - - - - - - - - - -		1.75638

Now in the triangle GEF we have FE, FG, and the angle FEG = HAE, to find FGE; thus,

As FG 50 - - - - - - - - -	Ar. Co.	8.30103
Is to FE 85 - - - - - - - - - - -		1.92942
So is sin. GEF 34° 6½′ - - - - - - - -		9.74877
To sin. FGE 72° 25′ - - - - - - - - -		9.97922

Finally, in ACE we have AE and the angles to find AC; thus,

As sin. ACE 73° 28½′ - - - -	Ar. Co.	0.01832
Is to sin. AEC 34° 6½′ - - - - - - - - -		9.74877
So is AE - - - - - - - - - - - -		1.75638
To AC 33.3785 - - - - - - - - - - -		1.52347

And BC = 85 — 33.3785 = 51.6215.

QUESTION 16.

Construction.

Make AC (Pl. 4, fig. 11.) = 50 = the given diagonal, and on it describe a semicircle ABC; make AE perpendicular to AC and $= \frac{1200}{50} =$ 24; draw EB parallel to AC, cutting the semicircle in B; join AB, BC, and draw CD and DA parallel to them; then will ABCD be the rectangle required.

Demonstration.

Since ABC is an angle in a semicircle, it is right, and ABCD is a rectangle. Also its area = AC × BF = 1200 perches = $7\frac{1}{2}$ acres.

Calculation.

FG = $\sqrt{BG^2 - BF^2} = \sqrt{49} = 7$; AF = AG—GF = 18, and AB = $\sqrt{AF^2 + FB^2} = \sqrt{900} = 30$, BC = $\sqrt{AC^2 - AB^2} = \sqrt{1600} = 40$.

QUESTION 17.

Construction.

Make AB (Pl. 4, fig. 12.) = the square root of the given area, and draw CE perpendicular to it; draw BC, making ABC = 30°, make AE = AC; bisect AC in D, and draw EF perpendicular to CE and = ED. Complete the parallelogram CEFG, which will be the one required.

Demonstration.

Since the angle B = 30°, and A = 90°, BC = 2 AC = CE = 4 CD, and EF = ED = 3 CD; therefore FC = $\sqrt{EF^2 + EC^2} = 5$ CD. Also $AB^2 = BC^2 - AC^2 = \frac{3}{4} BC^2 = \frac{3}{4} EC^2 =$ EC × ED = EC × EF = CEFG.

Calculation.

Since $AB^2 = \frac{3}{4} CE^2$, $CE^2 = \frac{4}{3} AB^2 = \frac{4}{3}$ the given area = 784, and CE = 28; hence EF = $\frac{3}{4}$ EC = 21.

QUESTION 18.

Construction.

With the given bearings and distances protract the figure ABCD, (Pl. 4, fig. 13.) and from B draw BP according to the given bearing

and distance of the spring. Produce DA and CB to meet in F, and through P draw EH parallel to AD. Bisect AF in G, join EG, and draw BM parallel to it, and MN parallel to FE. Make MT perpendicular to MN, and equal to the square root of the given area. Take MU a third proportional to MN and MT; draw UH parallel to MN, cutting AF in I; draw IK perpendicular to AF and equal to EP, and with the centre K and distance PH describe an arc cutting AD in Q; draw QPR, and the thing is done.

Demonstration.

In the similar triangles FGE and FMB, we have FB : FM :: FE : FG; therefore, 15.6, the triangle EFM = BFG; but EFM = ½ FMNE, and BFG = ½ BFA; hence FMNE = BFA. Again, because the triangles EPR, IQS, and PHS are similar, and the homologous sides EP (IK), IQ, and PH (KQ) form a right angled triangle, we have (Euclid 31. 6.) EPR+IQS=SPH. Add FISPE to each, and we have FQR = EFIH. But FBA = EFMN, hence BAQR = MNIH = MN.MU = MT^2 = the given area.

Calculation.

From the bearings of the lines the angles may be found as follow. AFB = BEP = 23°, ABF = 84° 30′, BAF = 72° 30′, EBP = 145° 30′, and EPB = 11° 30′. Then, in the triangle EBP we have all the angles and side BP, to find EP and EB;

Thus,

As sin. BEP 23°	Ar. Co.	0.40812
Is to sin. EBP 145° 30′		9.75313
So is BP 7.90		0.89763
To EP 11.452		1.05888

And,

As sin. BEP	Ar. Co.	0.40812
Is to sin. BPE 11° 30′		9.29966
So is BP		0.89763
To BE 4.031		0.60541

Also, in the triangle ABF, all the angles and side AB are given, to find BF and AF;

Thus,

As sin. AFB 23°	Ar. Co.	0.40812
Is to sin. BAF 72° 30′		9.97942
So is AB 15.20		1.18184
To BF 37.101		1.56938

And,

As sin. AFB	Ar. Co.	0.40812
Is to sin. ABF 84° 30′		9.99800
So is AB		1.18184
To AF 38.722		1.58796

And FE = FB—BE = 33.07, and FG = ½ AF = 19.361;

Also,

As EF 33.07	Ar. Co.	8.48057
Is to GF 19.361		1.28693
So is BF 37.101		1.56938
To FM 21.721		1.33688

Now, in the parallelogram MIHN, we have MN = FE = 33.07, and IMN = F = 23°, and the area = 100 square chains, to find MI;

Thus,

As MN, sin. IMN	MN 33.07	Ar. Co.	8.48057
	sin. IMN 23°	Ar. Co.	0.40812
Is to radius			10.00000
So is MIHN			2.00000
To MI 7.739			0.88869

Therefore PH = EH—EP = FM+MI—EP = 18.008. Now, in the right angled triangle IKQ, we have IK = EP = 11.452, and KQ = PH = 18.008, to find IQ; thus,

KQ+KI = 29.46	log.	1.46923
KQ—KI = 6.556		0.81664
		2)2.28587
IQ = 13.898		1.14293

Hence AQ = FQ—FA = FM+MI+IQ—FA = 4.636.

QUESTION 19.

Construction.

With the given bearings and distances, protract the figure ABCD, (Pl. 4, fig. 14;) then, by Prob. 17, Chap. IV. divide ABCD into two equal parts by the line EF, parallel to CD; also, by the same problem, divide ABCD, and EBAF, each into two equal parts by the lines OM and PN, parallel to AD; join MN, produce it to I, and draw OH parallel to IM; join IH, then will EF and IH be the division lines required.

Demonstration.

Because PN is parallel to OM, we have IN : NM :: IP : PO :: IG : GH, because NG is parallel to HM; therefore, PG is parallel to OH, and consequently to IM. Now since OH is parallel to IM, we have IHM = IOM, to each add AIMD, and AIHD = AOMD = ½ ABCD. In the same manner it may be shown that AIGF = ½ ABEF = ½ ABCD.

Calculation.

Draw EK and IL, each parallel to AD, and MU parallel to AB. From the given bearings find the angle A = 78° 30′, B = 139° 45′, C = 78° 45′, and D = 63°. By Prob. 17, Chap. IV., find EF and AF, thus,

As sin. C . sin. D	sin. C 78° 45′	Ar. Co.	0.00843
	sin. D 63° -	Ar. Co.	0.05012
Is to sin. A . sin. B	sin. A 78° 30′ - - - -		9.99119
	sin. B 139° 45′ - - -		9.81032
So is AB^2	AB 23 - - - - - -		1.36173
	AB - - - - - - - -		1.36173
To fourth term	383.282 - - - - - - -		2.58352
CD^2	2161.3201		
	2)2544.6021		
EF =	$\sqrt{1272.3010}$ = 35.67		

And,

As sin. P 38° 15′ - - - - - - -	Ar. Co.	0.20824
Is to sin. E 78° 45′ - - - - - - - - - -		9.99157
So is CD—EF 10.82 - - - - - - - -		1.03423
To FD - 17.14 - - - - - - - - - - -		1.23404
AD 49.64		
AF 32.50		

Then in the triangle ECK, we have the angles and side EK = FD, to find EC, thus,

As sin. C 78° 45′ - - - - - - -	Ar. Co.	0.00843
Is to sin. K 63° - - - - - - - - - -		9.94988
So is EK 17.14 - - - - - - - - - -		1.23404
To CE 15.57 - - - - - - - - - - -		1.19235

Consequently BE = BC—CE = 14.93. Now by the same problem find OM, AO, PN and AP, thus,

As sin. A . sin. D	sin. A - -	Ar. Co.	0.00881
	sin. D - -	Ar. Co.	0.05012
Is to sin. B . sin. C	sin. B - - - - - -		9.81032
	sin. C - - - - - -		9.99157
So is BC^2	BC 30.50 - - - - -		1.48430
	BC - - - - - - -		1.48430
To fourth term	675.18 - - - - - - -		2.82942
AD^2	2464.1296		
	2)3139.3096		

$$OM = \sqrt{1569.6548} = 39.62$$

And,

As sin. P 38° 30′ - - - - - - -	Ar. Co.	0.20585
Is to sin. D - - - - - - - - - - -		9.94988
So is AD—OM 10.02 - - - - - - - -		1.00087
To AO 14.34 - - - - - - - - - - -		1.15660

And,

As sin. A. sin. F	sin. A - - -	Ar. Co.	0.00881
	sin. F - - -	Ar. Co.	0.05012
Is to sin. B. sin. E	sin. B - - - - - -		9.81032
	sin. E - - - - - -		9.99157
So is BE^2	BE 14.93 - - - -		1.17406
	BE - - - - - - - -		1.17406
To Fourth term	161.79 - - - - -		2.20894

AF^2 - - - - 1056.25

2)1218.04

PN - - - $\sqrt{609.02} = 24.68$.

And,

As sin. P 38° 30′ - - - - - - -	Ar. Co.	0.20585
Is to sin. F - - - - - - - - - - - -		9.94988
So is AF — PN 7.82 - - - - - - - -		0.89321
To AP 11.19 - - - - - - - - - - -		1.04894

Hence OP = AO — AP = 3.15; wherefore we have

OM—PN (14.94) : PN (24.68) :: OP (3.15) : IP = 5.20; and AI = AP—IP = 5.99.

In the triangle MUL we have the angle U = A, L = D, and side MU = IO = IP + PO = 8.35, to find ML and UL; thus,

As sin. ULM 63° - - - - - - -	Ar. Co.	0.05012
Is to sin. MUL 78° 30′ - - - - - -		9.99119
So is MU 8.35 - - - - - - - - - -		0.92169
To ML 9.18 - - - - - - - - - - - -		0.96300

And,

As sin. MLU 63° - - - - - - -	Ar. Co.	0.05012
Is to sin. UML 38° 30′ - - - - - - -		9.79415
So is MU 8.35 - - - - - - - - - -		0.92169
To UL 5.83 - - - - - - - - - - - -		0.76596

Therefore IL = IU+UL = OM+UL = 45.45, and from the similar triangles ILM and OMH, we have

As IL (45.45) : LM (9.18) :: OM (39·62) : MH = 8.

Now, in the triangle ILH, we have the angle L = D, and sides IL and LH = LM+MH = 17.18, to find the angle LIH; thus,

As LI+LH	62.63	Ar. Co.	8.20322
Is to LI—LH	28.27		1.45133
So is tang. $\frac{LHI+LIH}{2}$	58° 30′		10.21268
To tang. $\frac{LHI-LIH}{2}$	36 23		9.86723
LIH	22 7		
Bearing IL	66 15		
" IK	S. 88 22 E.		

THE END.

PLATE 1

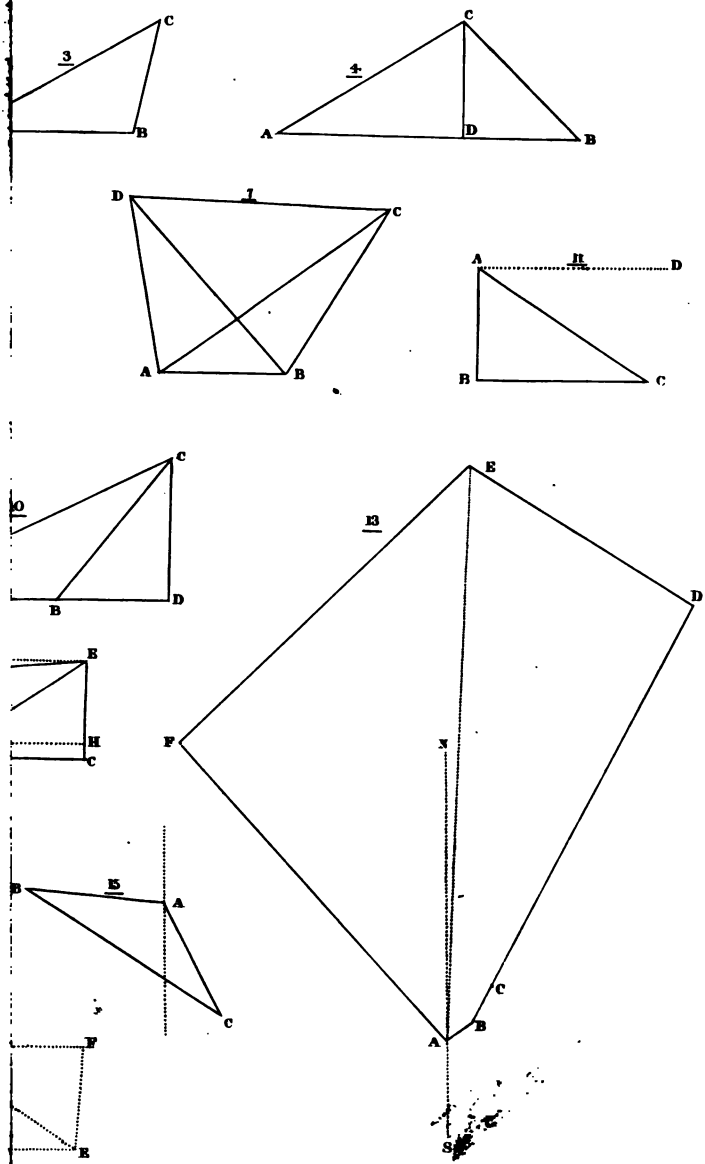


PLATE 2

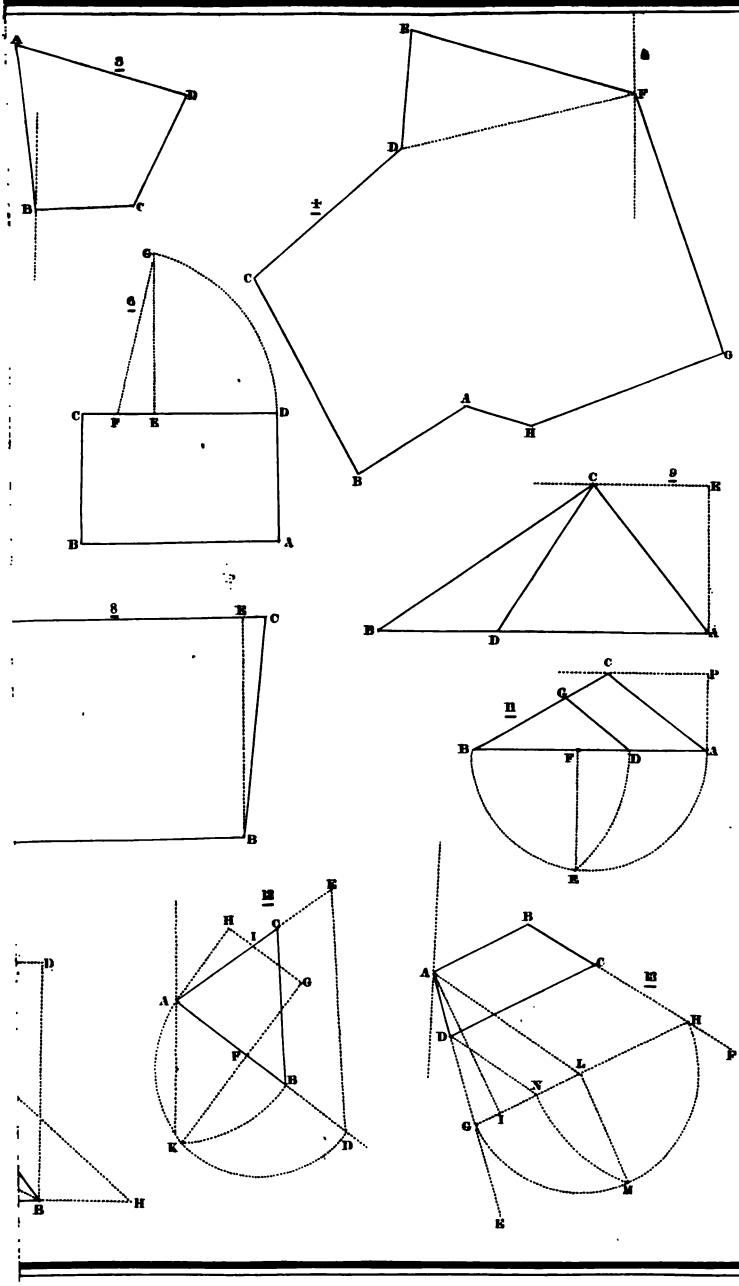
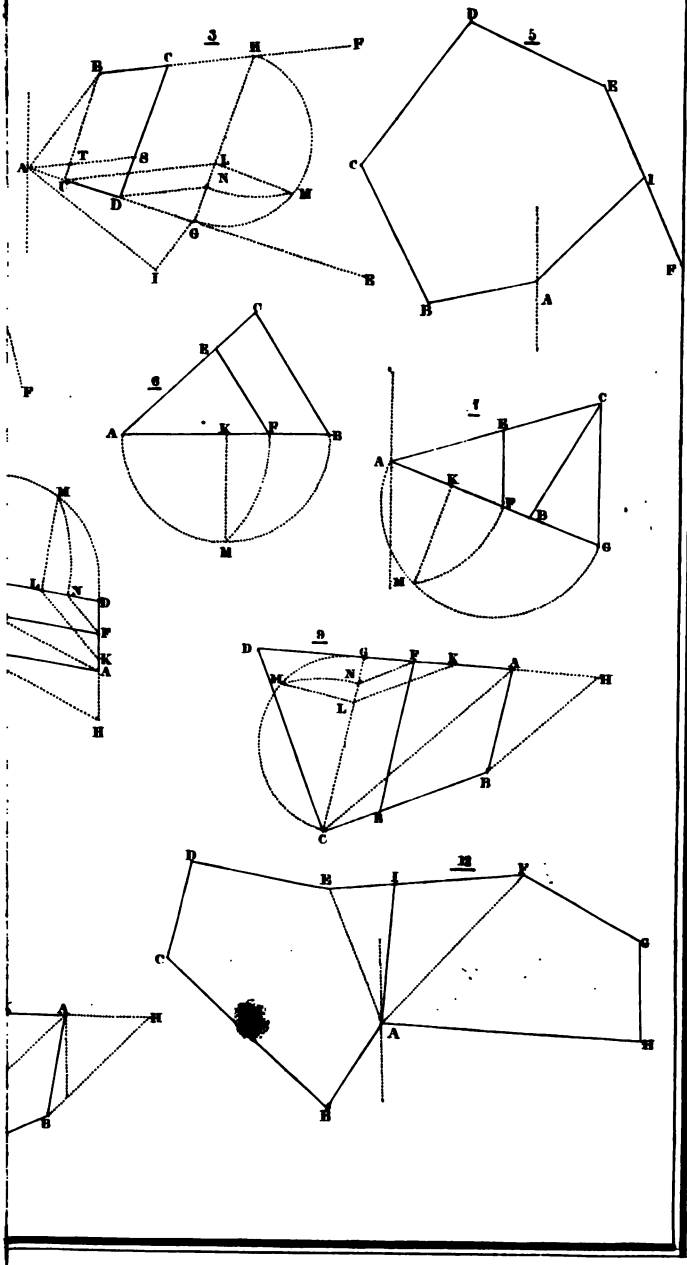


PLATE 3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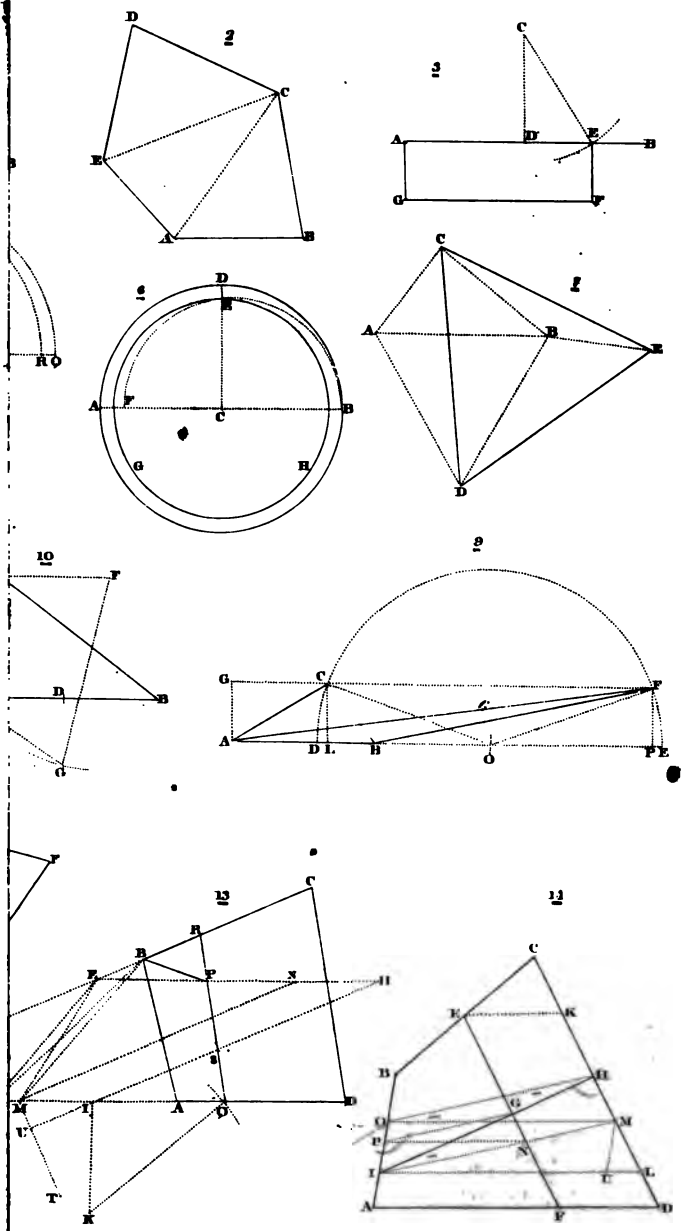
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A very important division of the second part is that exhibiting the derivation of several thousand English words from their Latin and Greek roots, arranged and defined as in the following example:

Caput, capitis, the head.

Cap'ital. Chief, principal.
Capita'tion. Counting by heads.
Cape. A headland.
Cap'tain. A chief commander.
Chap'ter. A division or head.
Decap'itate. To behead.
Oc'ciput. The hinder part of the head.
Precip'itate. *v.* To tumble headlong, to hurry.
Precip'itate. *a.* Headstrong, hasty.
Precip'itate. *n.* A sediment.
Prec'ipice. A headlong steep.
Recapit'ulate. To repeat the heads of a discourse.

To those pupils who are not to receive a classical education, this part of the work will be of great service. It will show them how many of the important words in their own language, are derived from those of Greece and Rome, and give them a much clearer idea of the meaning of such words, than they would otherwise acquire. Even those who are subsequently to study the Latin and Greek will find their labour lightened by their previous knowledge of many of the most useful words in those languages.